Doctor, how good is that test?
A primer on predictive value

Disease Detection via Diagnostic Testing

Suppose we are interested in detecting the presence of disease X based on some diagnostic test Z.

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Type of Question to be Considered

A 75 year old female (Mary) without symptoms has a screening mammogram at a center that reports an historical test sensitivity of 80% and a specificity of 90%.

Mary's test turns out to be positive.

What is the chance that Mary has Breast Cancer?

Disease Detection via Diagnostic Testing

There are three basic things that we need to know first

1. The sensitivity of diagnostic test Z
2. The specificity of diagnostic test Z
3. The prevalence of disease X
(in the population to which our patient belongs)

Disease Detection via Diagnostic Testing

There are three basic things that we need to know first

1. **The sensitivity of diagnostic test Z**
2. **The specificity of diagnostic test Z**
3. **The prevalence of disease X**
   (in the population to which our patient belongs)

Sensitivity: Probability that Z detects X when X is present ('sick')
Specificity: Probability that Z does not detect X when X is not present ('well')
Prevalence: Probability of Mary having X prior to testing

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PREVALENCE

Probability that Mary has X (prior to test)

Depends on frequency that X exists within the at-risk population

'Prior Probability'

↓

$$\text{Prevalence} = \frac{\text{N with X}}{\text{N at-risk for X}}$$

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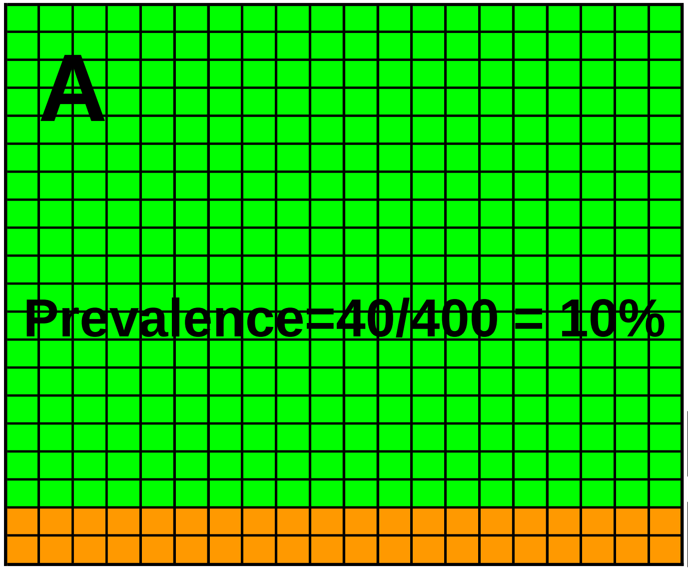
All US Adults N= 200,000,000



PREVALENCE

Probability that Mary has X (prior to test)

Depends on frequency that X exists within the at-risk population

'Prior Probability'

Don't have X

Do have X = 40

Each box = 500,000 adults
Total boxes = 20 x 20 =400
All at-risk for X

$$\text{Prevalence} = \frac{\text{N with X}}{\text{N at-risk for X}}$$

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PREVALENCE

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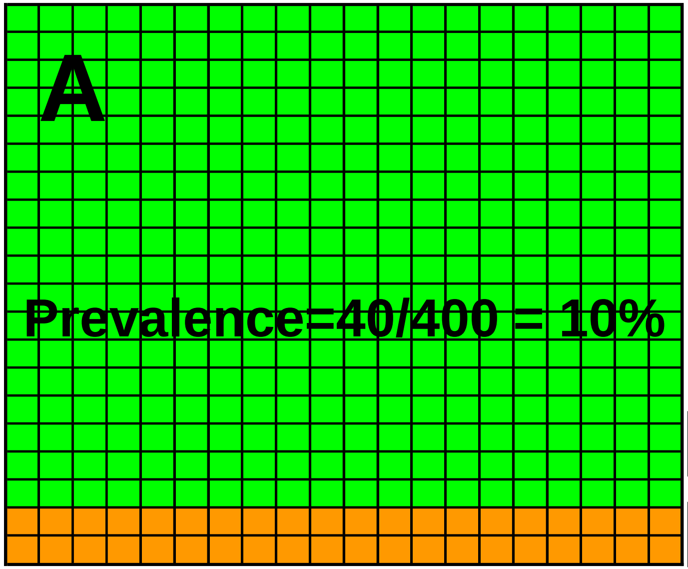

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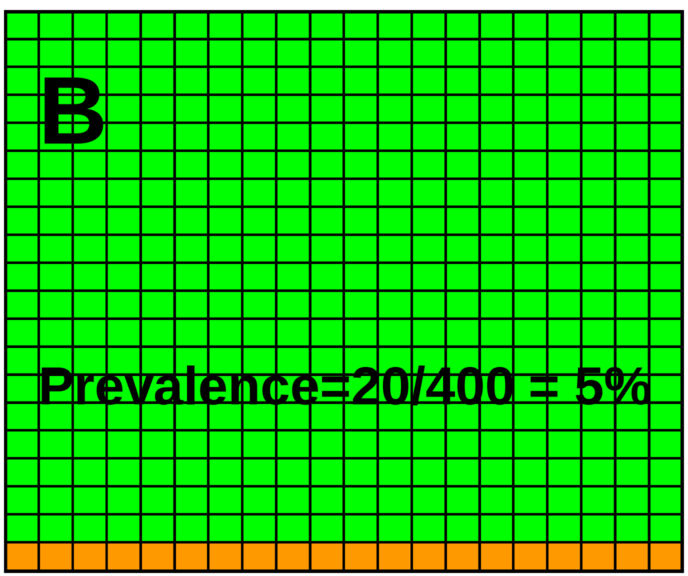

Don't have X

Do have X = 20

Probability that Mary has X (prior to test)

Depends on frequency that X exists within the at-risk population

'Prior Probability'

$$\text{Prevalence} = \frac{\text{N with X}}{\text{N at-risk for X}}$$

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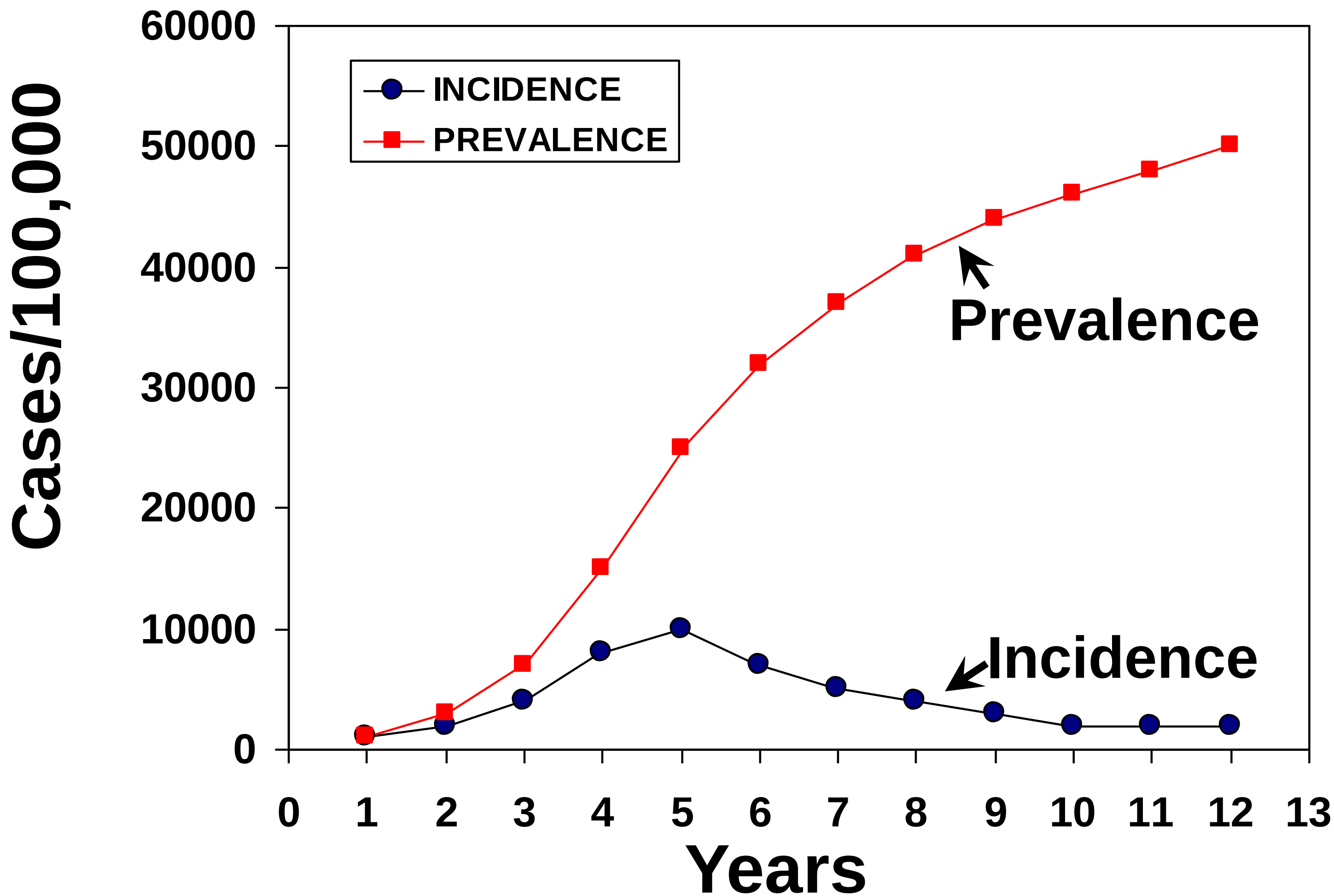
Prevalence vs. Incidence

All Cases NOW — **New Cases/Year**



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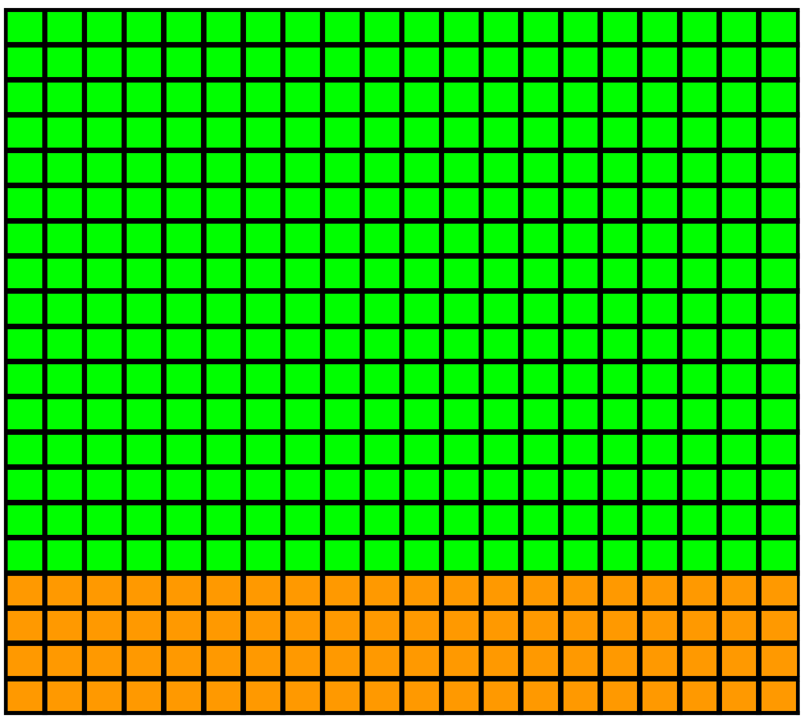
Hypothetical Research Study

Evaluate the Sensitivity and Specificity of a Diagnostic Test Z

Adults N=400
Each box = 1 adult



Facts

- Don't have disease X
- Do have disease X

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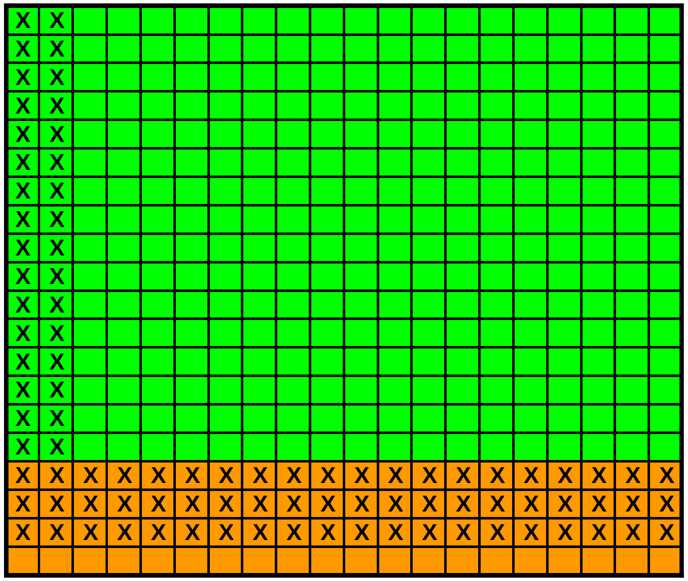
Study Results

Research Study Adults N=400
Each box = 1 adult



Don't have X

Do have X

X Do have X and detected by test Z (TP)

X Don't have X but 'detected' by test Z (FP)

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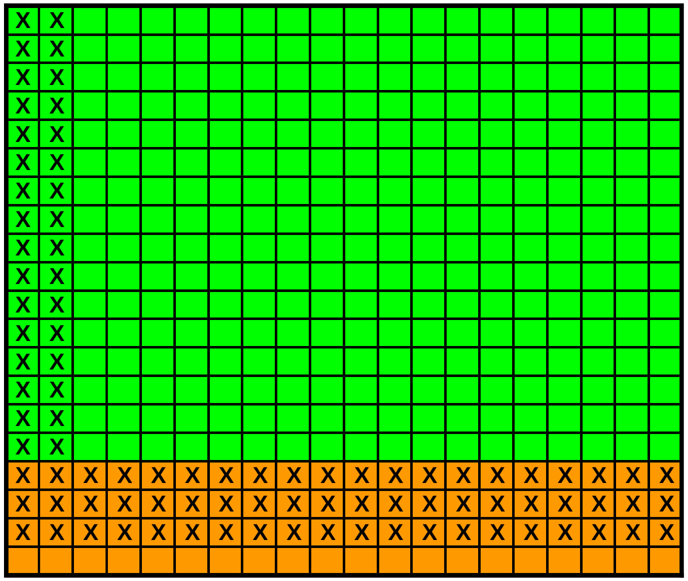
Research Study Adults N=400
Each box = 1 adult



Study Results

Don't have X

Do have X

X Do have X and detected by test Z (TP)

X Don't have X but 'detected' by test Z (FP)

SENSITIVITY

Probability that test Z detects X when in fact X is present

$$\frac{60}{80}$$

60/80 = 75%

SPECIFICITY

Probability that test Z does not detect X when X is not present

$$\frac{320 - 32}{320}$$

288/320 = 90%

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Possible Results of a 'Diagnostic' Test

FP → Detects something that does NOT exist
FN → Does NOT detect something that DOES exist
TP → Detects something that DOES exist
TN → Does NOT detect something that does NOT exist

FP → False Positive FN → False Negative
TP → True Positive TN → True Negative

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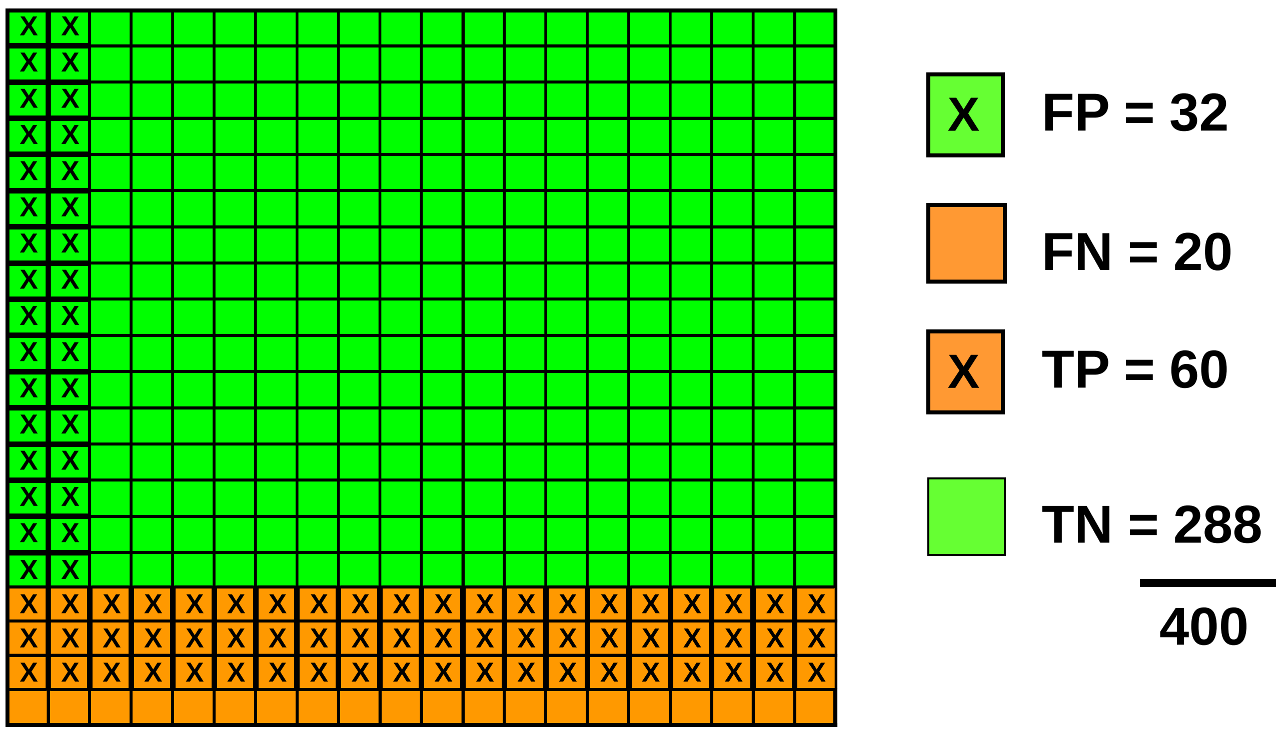

Features of a ‘Diagnostic’ Test Itself

$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

$$\text{Sensitivity} = \frac{60}{60 + 20}$$

$$= 75\%$$

$$\text{Specificity} = \frac{TN}{TN + FP}$$

$$\text{Specificity} = \frac{288}{288 + 32}$$

$$= 90\%$$

***High Sensitivity* Implies that:**
***If condition is present* test has high probability of so indicating**

The null hypothesis (NH) for this is: *‘patient has the condition’* so has low type I error (α) → Low probability of rejecting the NH when NH is TRUE

***High Specificity* Implies that:**
***If condition is not present* test has high probability of so indicating**

The null hypothesis (NH) also is: *‘patient has the condition’* so has low type II error (β) → High probability of rejecting the NH when NH is FALSE

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Predictive Values of a ‘Diagnostic’ Test

What is the probability that a person with a *positive* test actually has the condition?

Positive Predictive Value (PPV) of the diagnostic test as applied to the person

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What is the probability that a person with a *positive* test actually has the condition?

Positive Predictive Value (PPV) of the diagnostic test as applied to the person

$$PPV = \frac{\text{Sensitivity} \times \text{Prevalence}}{\text{Sensitivity} \times \text{Prevalence} + (1-\text{Specificity})(1-\text{Prevalence})}$$

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Predictive Values of a 'Diagnostic' Test

What is the probability that a person with a *positive* test actually has the condition?

<u>Positive Predictive Value</u> (PPV) of the diagnostic test as applied to the person

$$\text{PPV} = \frac{\text{Sensitivity x Prevalence}}{\text{Sensitivity x Prevalence + (1-Specificity)(1-Prevalence)}}$$

$$\text{PPV} = \frac{0.75 \times 0.1}{0.75 \times 0.1 + (1-0.9)\ (1-0.1)} = 45.5\%$$

A positive test would mean there is less than a 50% chance she has the condition

Predictive Values of a ‘Diagnostic’ Test

What is the probability that a person with a *negative* test does NOT have the condition?

<u>Negative Predictive Value</u> (NPV) of the diagnostic test as applied to the person

$$\text{NPV} = \frac{\text{Specificity x (1 – Prevalence)}}{\text{Specificity (1-Prevalence) + Prevalence (1-Sensitivity)}}$$

$$\text{NPV} = \frac{0.90 \text{ x } (1 - 0.1)}{0.90 \text{ x } (1 - 0.1) + (0.1)\,(1 - 0.75)} = 97\%$$

A negative test would mean there is 97% chance that she does not have the condition

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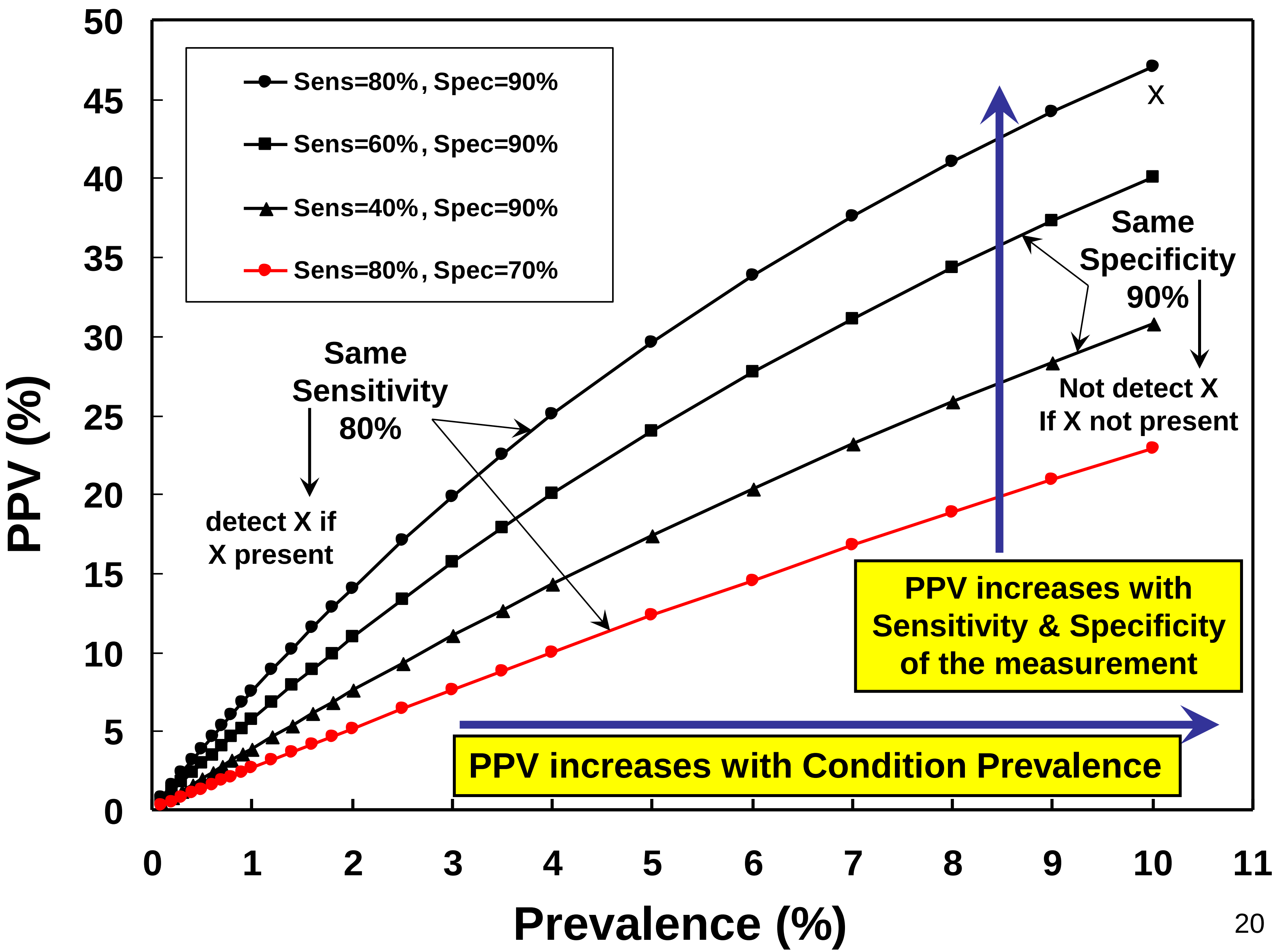

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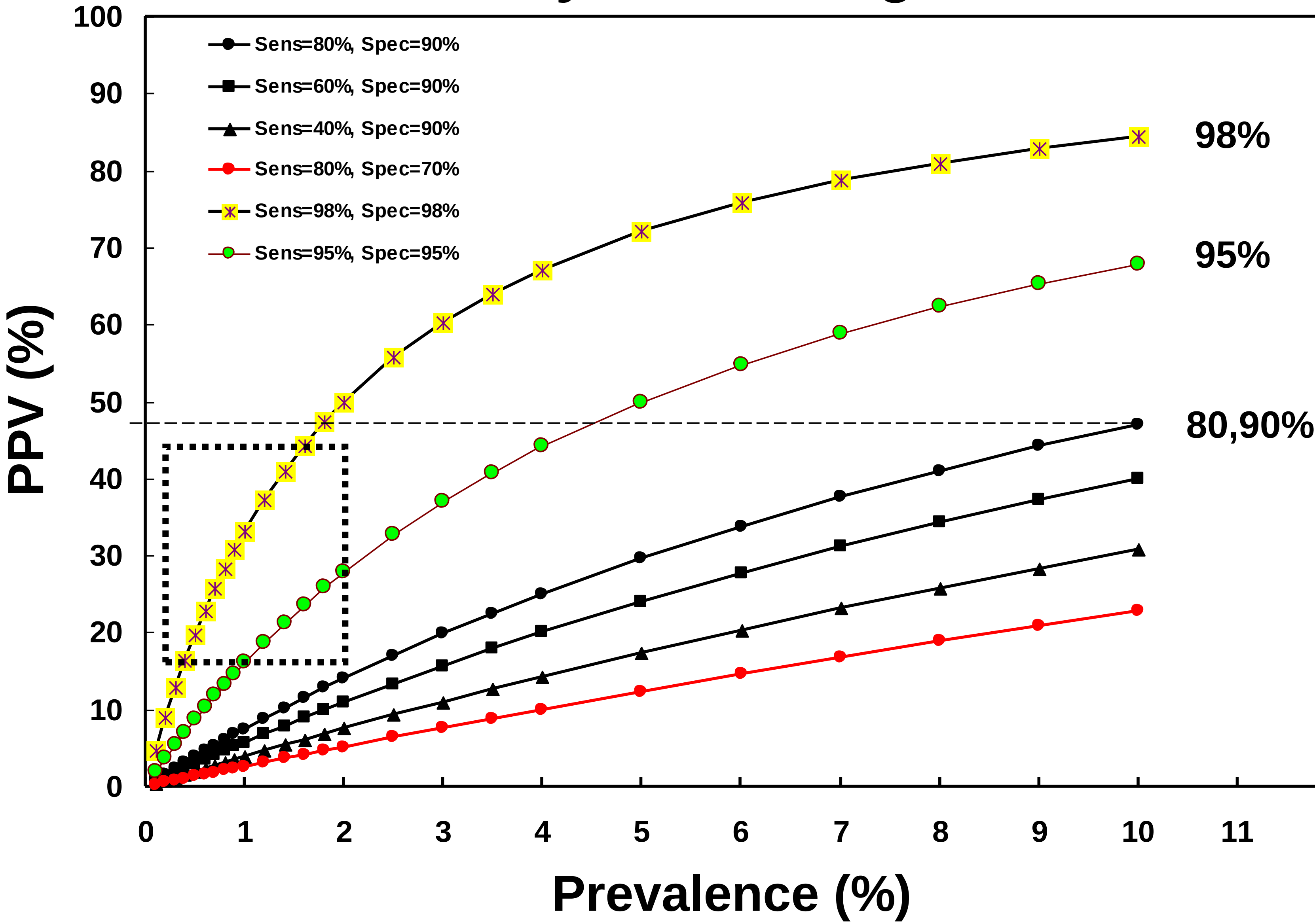
At low prevalence PPV falls dramatically even at high S & S!



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PPV Example – Screen Mammograms

$$PPV = \frac{\text{Sensitivity x Prevalence}}{\text{Sensitivity x Prevalence + (1-Specificity)(1-Prevalence)}}$$

What is the probability that a white women age 55 who has a positive screening test actually has breast cancer?

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What are sensitivity and specificity of mammography?

Kavanagh AM, Giles GG, Mitchell H, Cawson JN. The sensitivity, specificity, and positive predictive value of screening mammography and symptomatic status. Journal of medical screening 2000;7(2):105-110.

- **100,000 + women >=40 years of age**
- **Asymptomatic first screens no family Hx of BC**

Sensitivity: 75.6% → 75%
Specificity: 94.9% → 95%

PPV Example – Screen Mammograms

$$PPV = \frac{\text{Sensitivity x Prevalence}}{\text{Sensitivity x Prevalence + (1-Specificity)(1-Prevalence)}}$$

What is the probability that a white women age 55 who has a positive screening test actually has breast cancer?

Sensitivity = 75% Specificity = 95% Prevalence=?

↑

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Estimated prevalence percent [a] on January 1, 2006, of the SEER 11 population diagnosed in the previous 16 years
By Age at Prevalence, Race/Ethnicity and Sex

2.3%

Race/Ethnicity	Sex	Age Specific (Crude)										Age-Adjusted [f]
		All Ages	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80+	All Ages
All Races [c]	Both Sexes	0.5773%	-	0.0002%	0.0057%	0.0814%	0.4260%	1.1244%	1.9183%	2.4904%	2.8998%	0.5857%
	Males	0.0063%	-	-	-	0.0004%	0.0023%	0.0093%	0.0242%	0.0412%	0.0550%	0.0075%
	Females	1.1379%	-	0.0004%	0.0116%	0.1647%	0.8490%	2.1882%	3.6274%	4.3807%	4.4781%	1.0696%
White [c]	Both Sexes	0.6330%	-	0.0002%	0.0053%	0.0808%	0.4294%	1.1649%	2.0317%	2.6780%	3.0991%	0.6162%
	Males	0.0068%	-	-	-	0.0002%	0.0022%	0.0091%	0.0241%	0.0435%	0.0591%	0.0077%
	Females	1.2596%	-	0.0004%	0.0110%	0.1671%	0.8699%	2.3024%	3.8921%	4.7584%	4.7661%	1.1365%
Black [c]	Both Sexes	0.3941%	-	-	0.0078%	0.0962%	0.4214%	0.9915%	1.6239%	2.0368%	2.3114%	0.5063%
	Males	0.0061%	-	-	-	-	0.0036%	0.0132%	0.0415%	0.0448%	0.0473%	0.0093%
	Females	0.7463%	-	-	0.0150%	0.1797%	0.7883%	1.8049%	2.8559%	3.3621%	3.4078%	0.8709%
Asian/ Pacific Islander [c]	Both Sexes	0.4222%	-	-	0.0059%	0.0726%	0.4106%	0.9699%	1.3902%	1.5756%	1.6294%	0.4308%
	Males	0.0034%	-	-	-	-	-	0.0067%	0.0106%	0.0230%	0.0266%	0.0040%
	Females	0.8155%	-	-	0.0118%	0.1404%	0.7865%	1.8153%	2.5596%	2.7136%	2.6856%	0.7801%
Hispanic [d]	Both Sexes	0.2191%	-	-	0.0046%	0.0554%	0.2939%	0.7737%	1.2684%	1.6101%	1.7243%	0.3870%
	Males	0.0013%	-	-	-	-	0.0010%	0.0048%	0.0081%	0.0152%	0.0200%	0.0029%
	Females	0.4495%	-	-	0.0100%	0.1191%	0.6046%	1.5030%	2.3360%	2.7707%	2.7149%	0.7060%

PPV Example – Screen Mammograms

$$PPV = \frac{\text{Sensitivity x Prevalence}}{\text{Sensitivity x Prevalence + (1-Specificity)(1-Prevalence)}}$$

What is the probability that a white women age 55 who has a positive screen test actually has breast cancer?

Sensitivity = 75% Specificity = 95% Prevalence = 3.2%

PPV = 26.1%

There is about a 1 in 4 chance that she HAS BC

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NPV Example – Screen Mammograms

$$NPV = \frac{\text{Specificity x (1 – Prevalence)}}{\text{Specificity (1-Prevalence) + Prevalence (1-Sensitivity)}}$$

What is the probability that a white women age 55 who has a negative screen test does NOT have breast cancer?

Sensitivity = 75% Specificity = 95% Prevalence = 3.2%

NPV = 99.4%

There is a 99.4% chance that she does NOT have BC

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Individual Class Problem – Turn in with all calculations

A 75 year old female (Mary) without symptoms has a screening mammogram at a center that reports an historical test sensitivity of 80% and a specificity of 90%.

Mary's test turns out to be positive.

What is the chance that Mary has BC?

Questions or Comments?