

# **Spectral Analysis and Heart Rate Variability: Principles and Biomedical Applications**

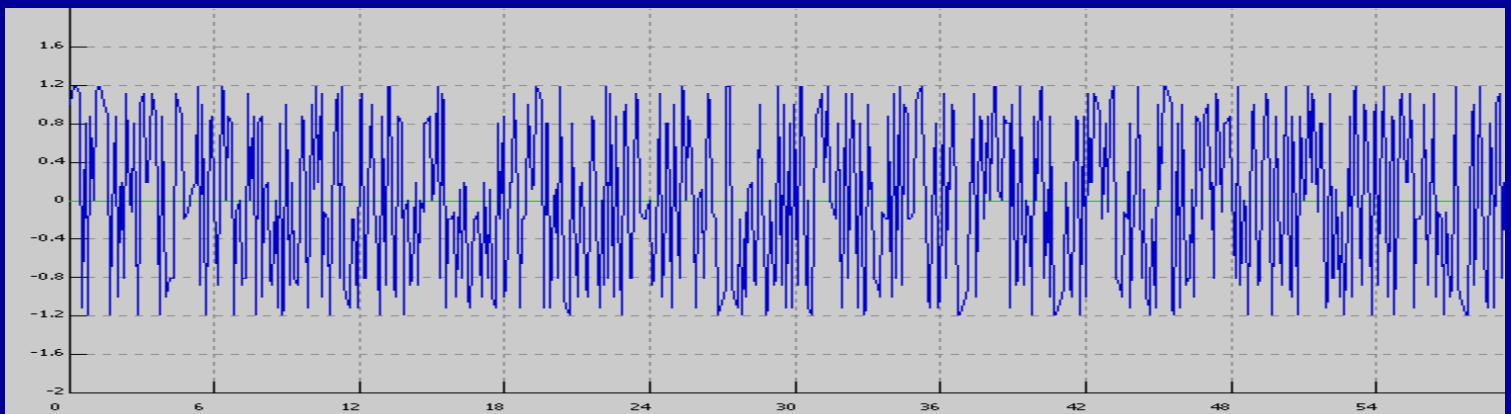
**Dr. Harvey N. Mayrovitz**

# **Why Spectral Analysis?**

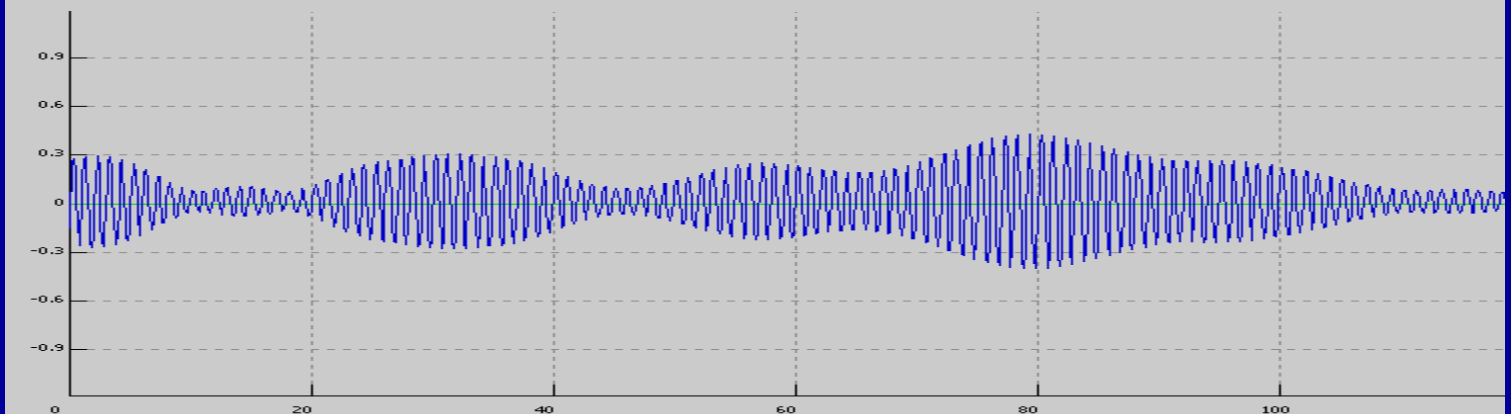
**Detection and characterization of cyclical or periodic processes present in physiological signals**

**Rhythms are present in nearly all physiological signals - but not always evident to the 'naked eye'!**

**Signal**



**Filtered**



**Spectrum**

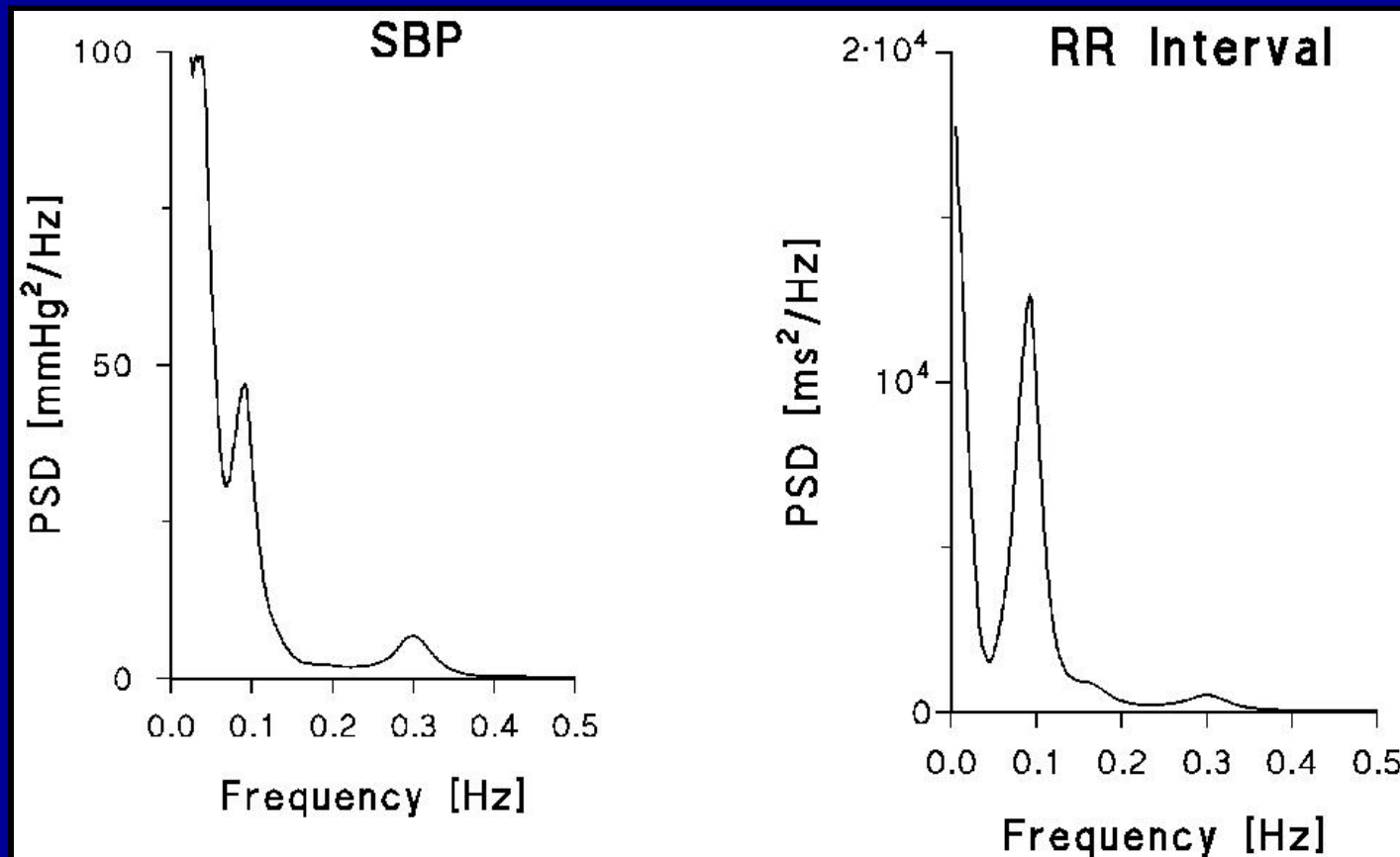


**How do you  
extract spectral (frequency)  
components present in  
physiological signals?**

# Power Spectral Density

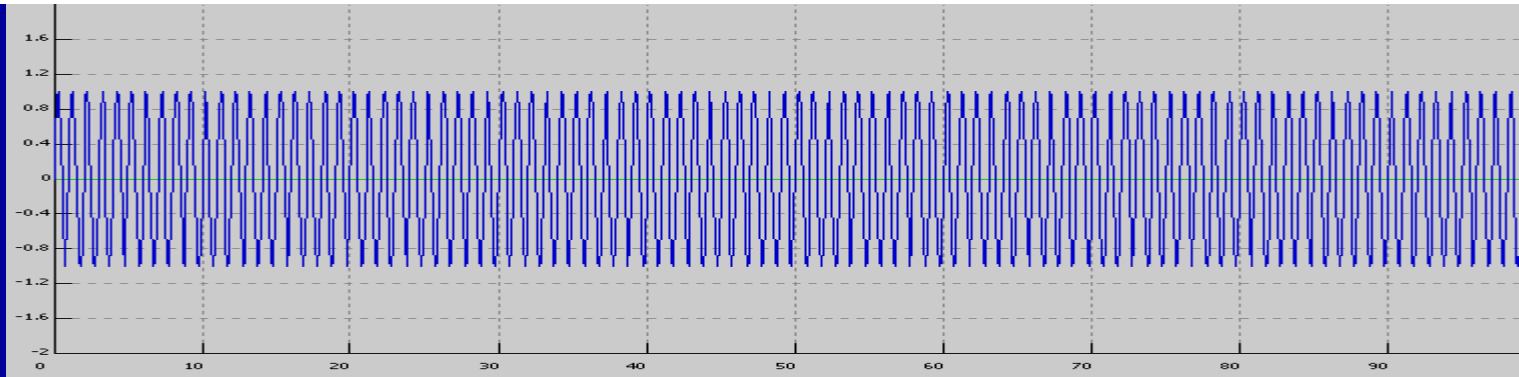
Amount of power per unit (density) of frequency (spectral)  
as a function of frequency

PSD describes how the power (or variance) of a  
time series is distributed with frequency!

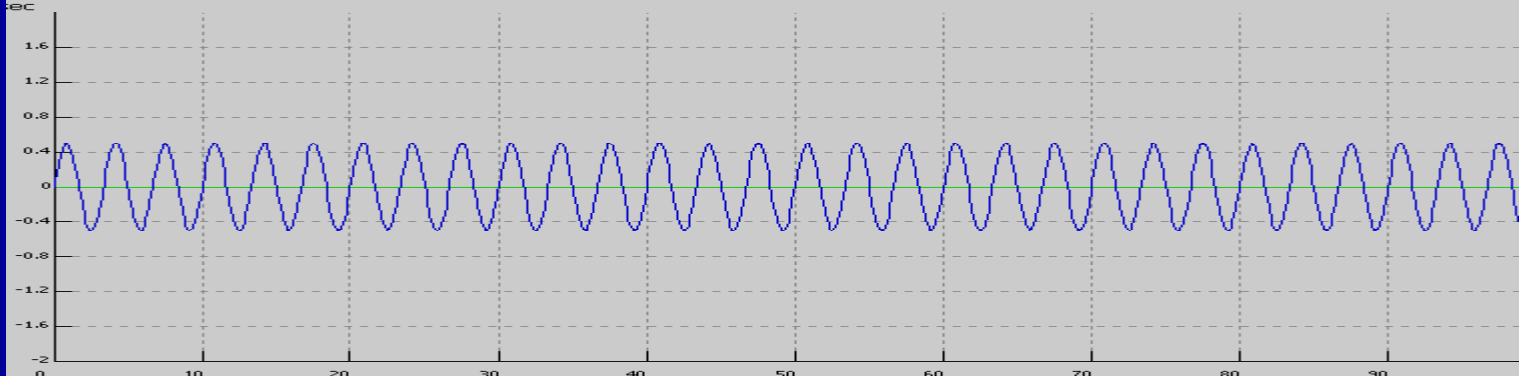


# Example with Simulated Signals

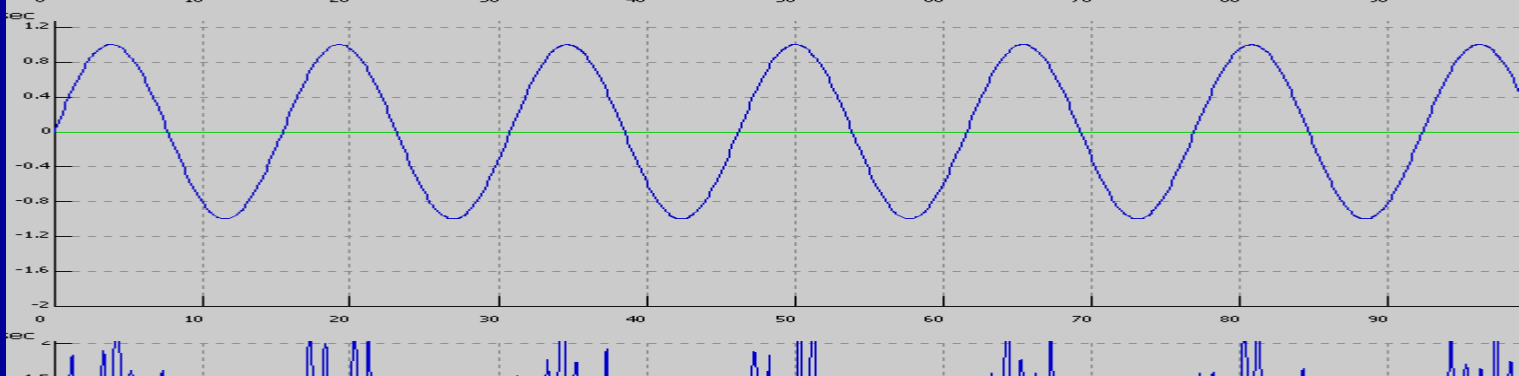
**A**  
**1.0 Hz**



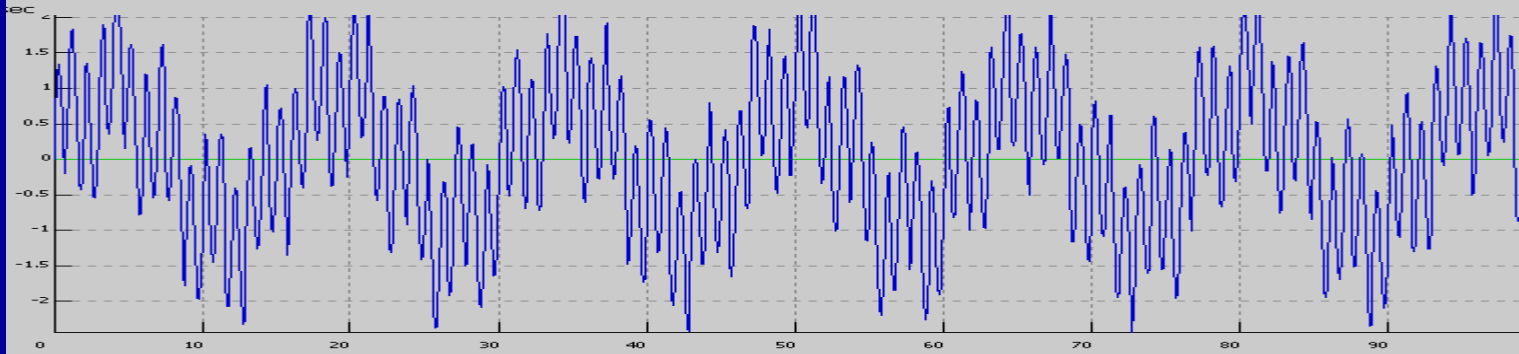
**B**  
**0.3 Hz**



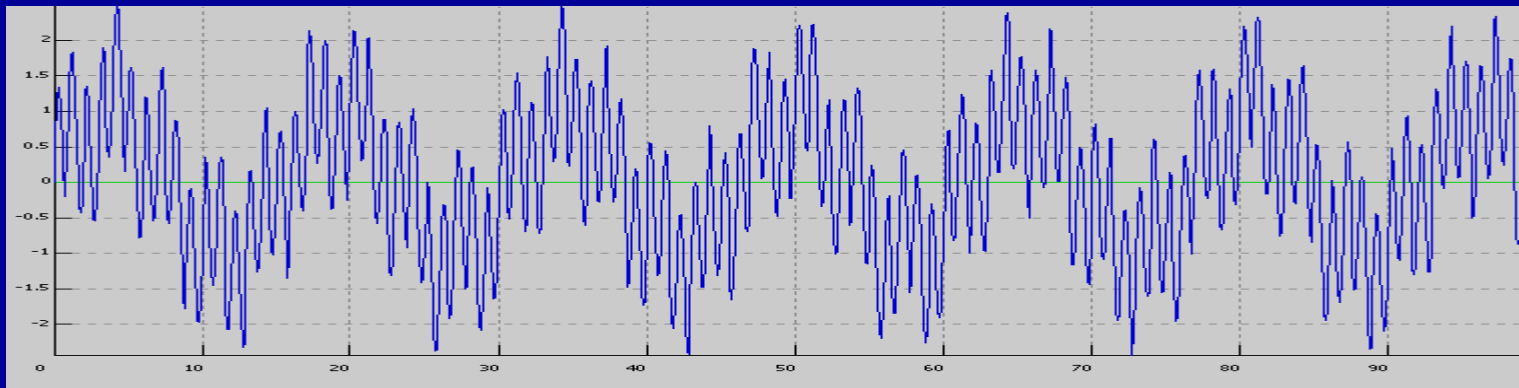
**C**  
**0.065 Hz**



**A+ B+ C**

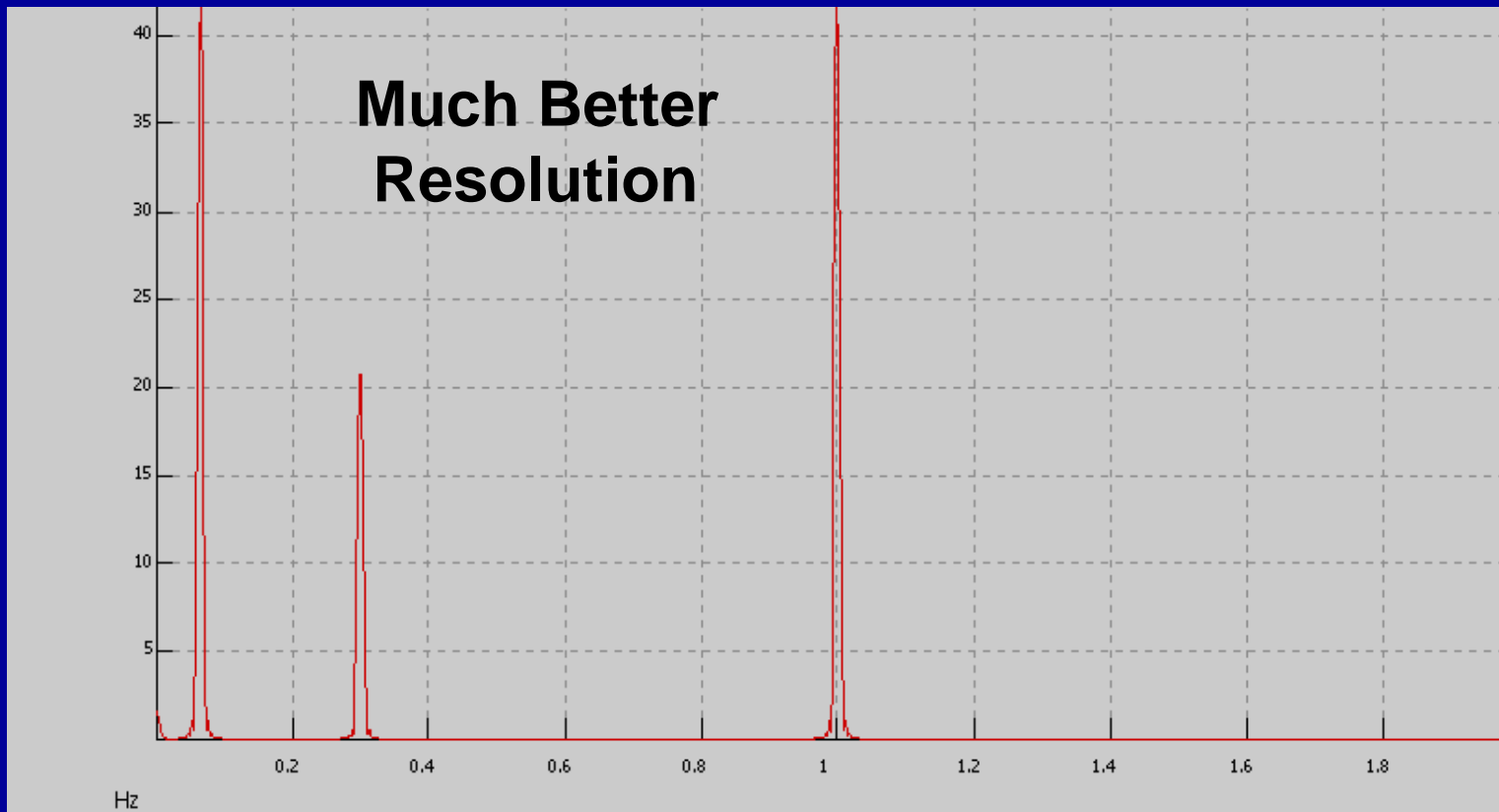
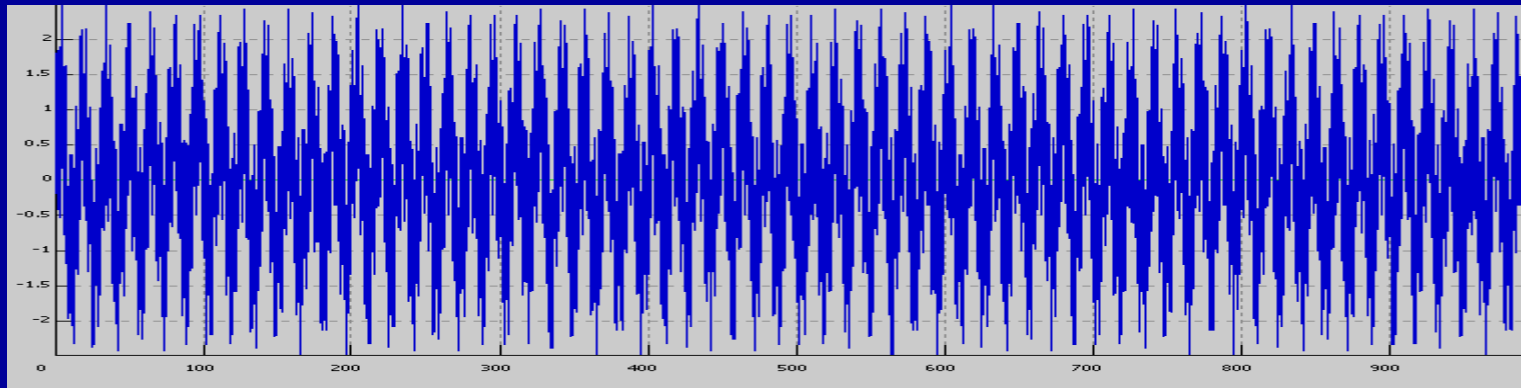


**A+ B+ C**  
**100 sec**



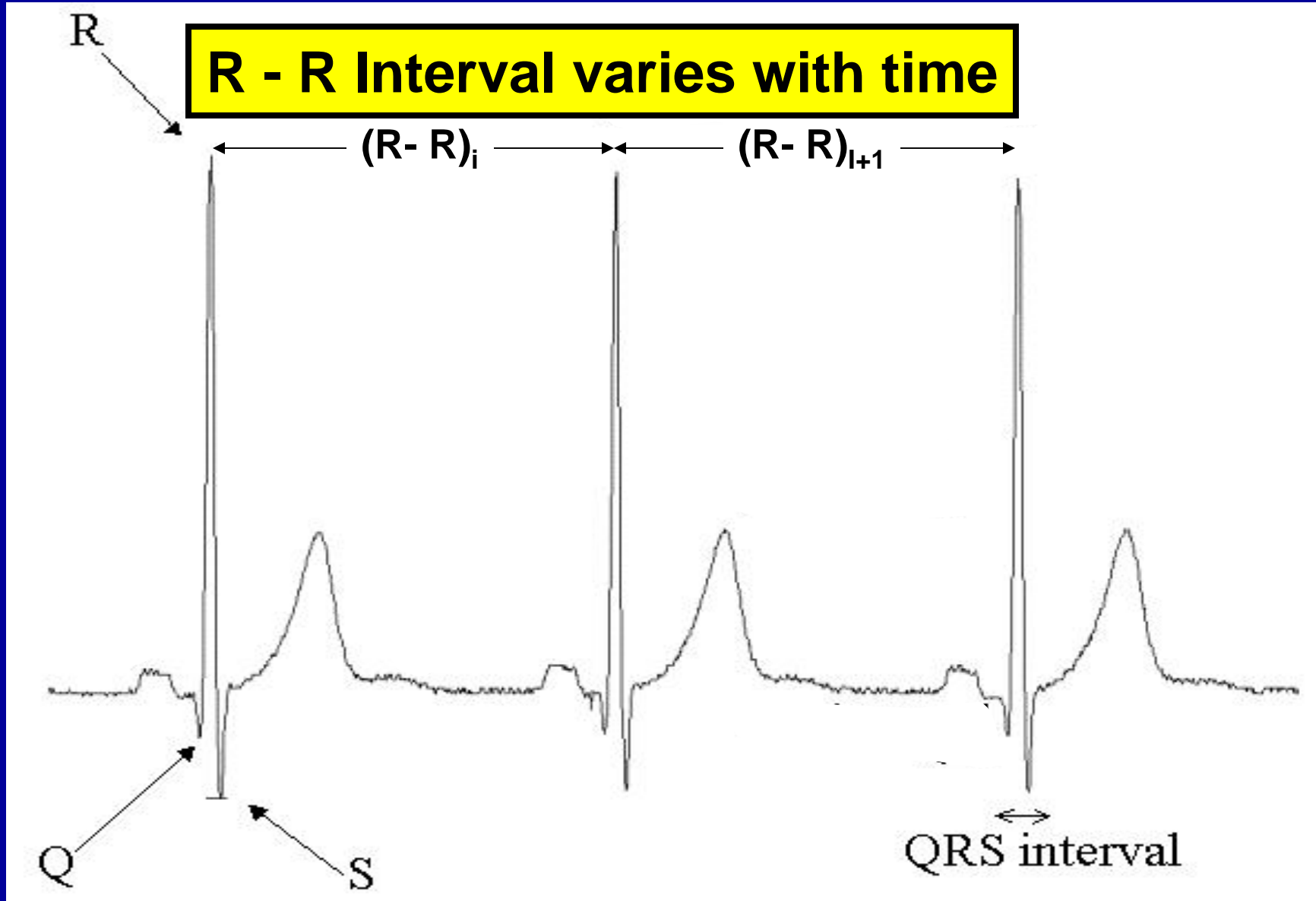


**A+ B+ C**  
**1000 sec**

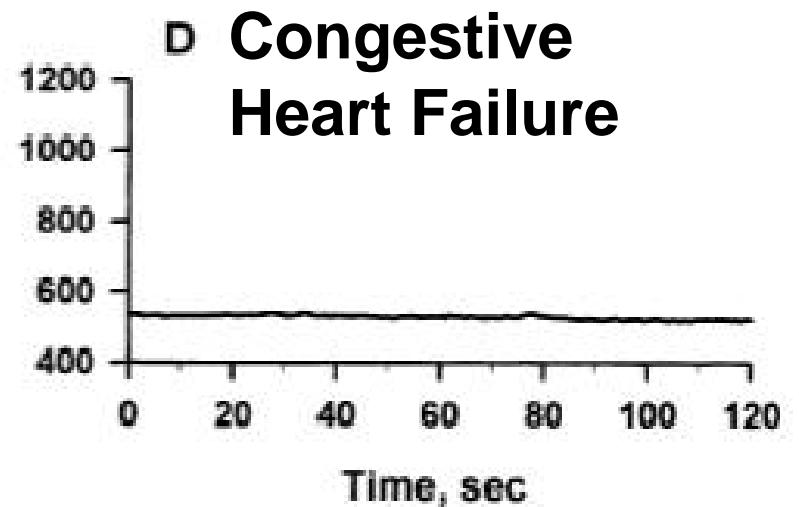
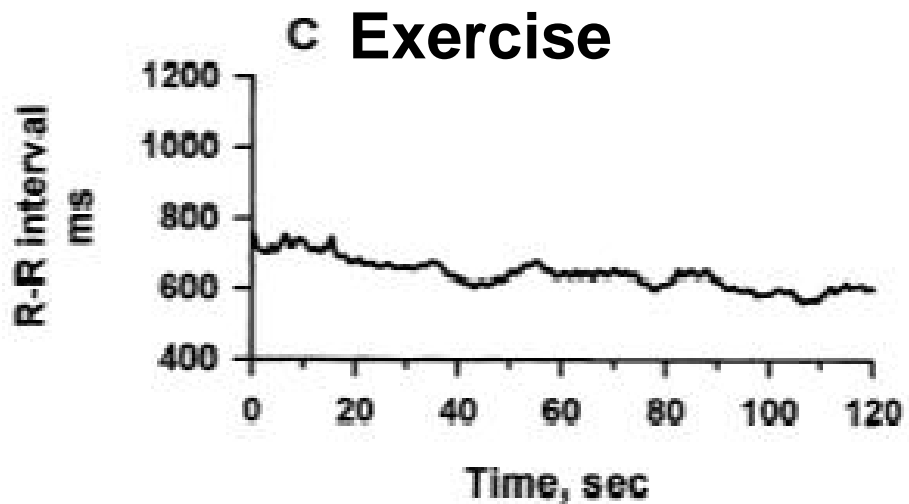
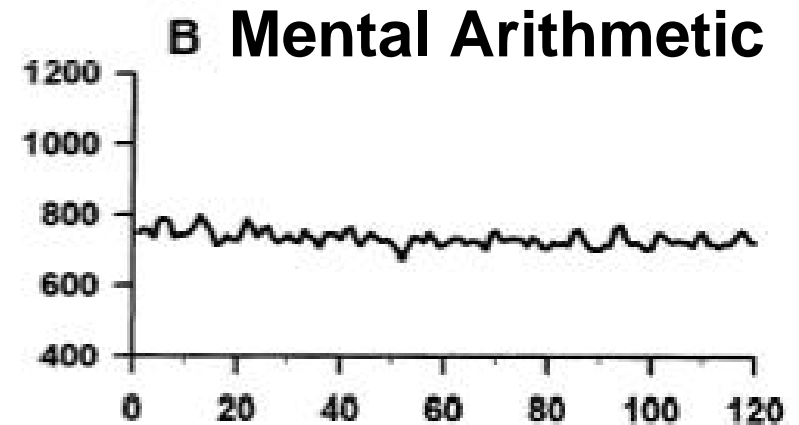
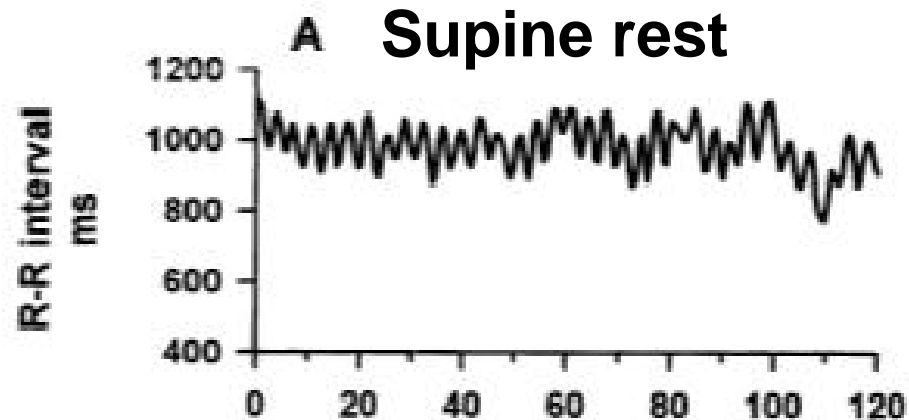


# **Generating a time series signal from the Electrocardiogram**

# R- R Time Series

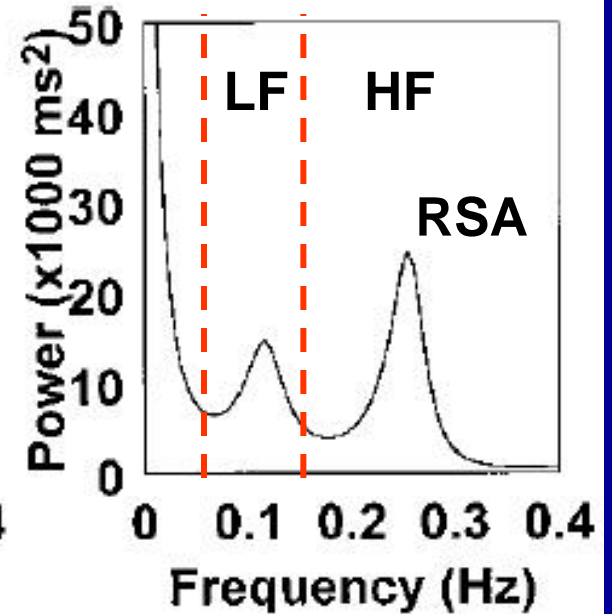
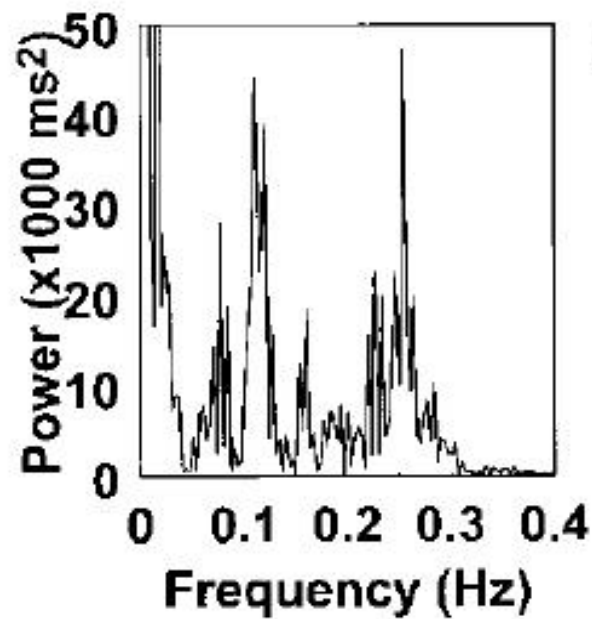
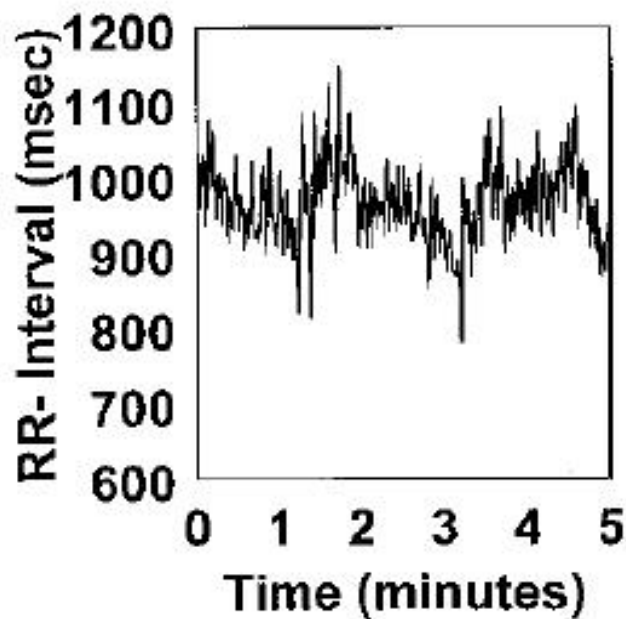


# R- R Time Series



# Heart Rate Variability

# Heart Rate Variability (HRV)



# Heart Rate Variability (HRV)

Peripheral Vascular  
& Thermoregulatory



**ULF: <0.003 Hz**

Baroreceptors phase delay  
Sympathetic & Parasympathetic



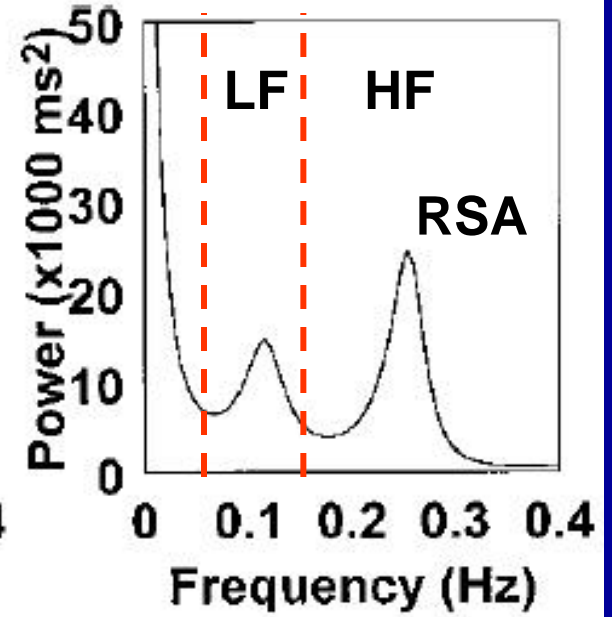
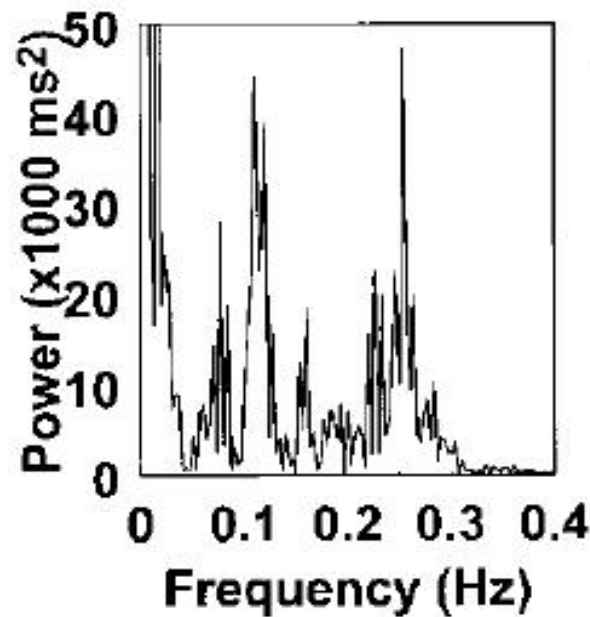
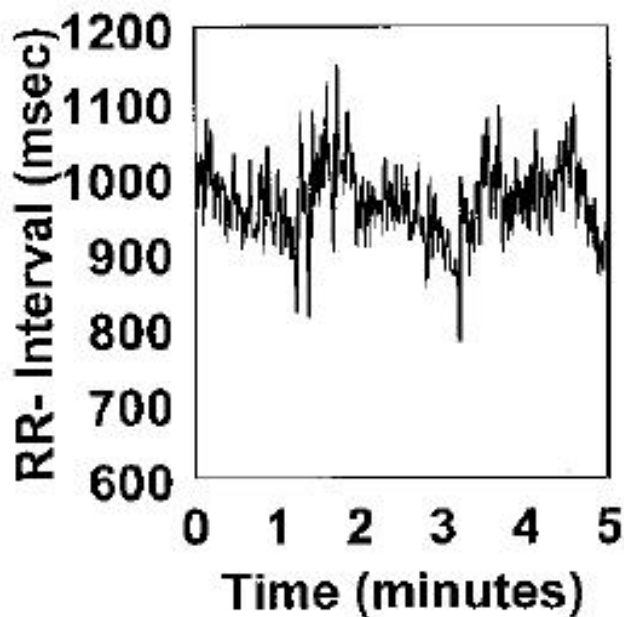
**VLF: 0.003 - 0.04 Hz**

**LF: 0.04 - 0.15 Hz**

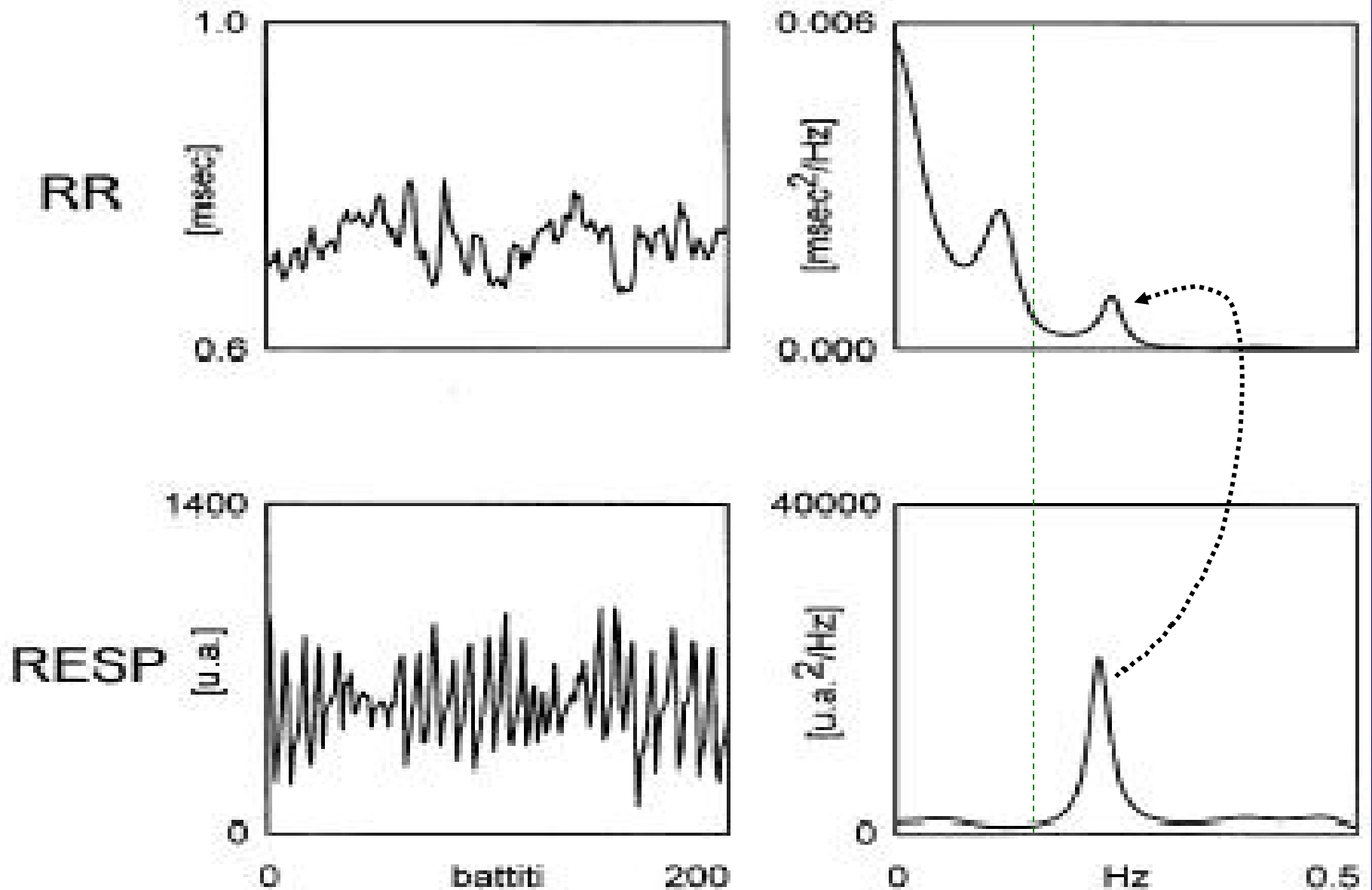
Respiratory Sinus Arrhythmia (RSA)  
Cardiac Vagal Activity Change



**HF: 0.15 - 0.40\* Hz**

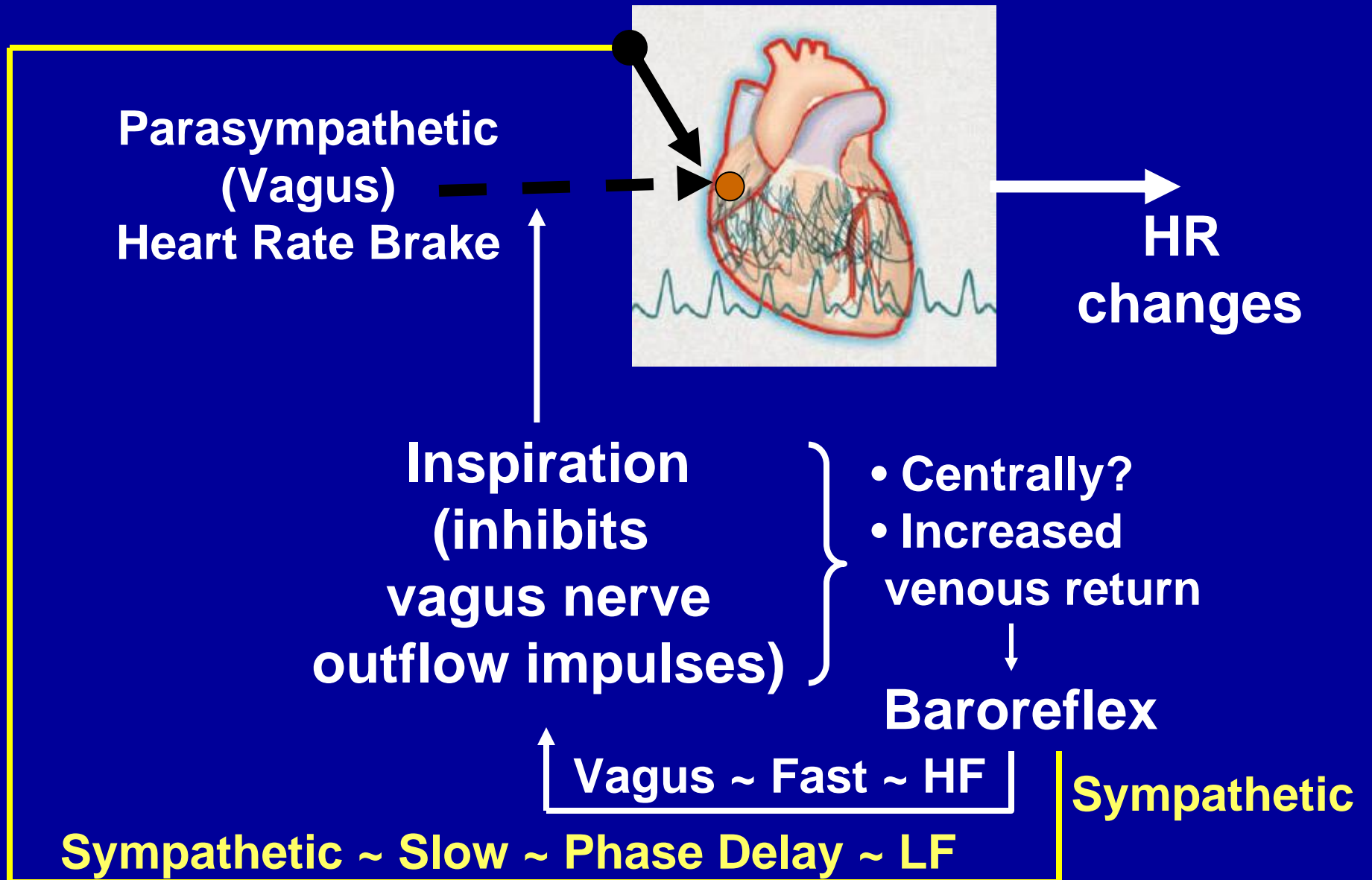


# RSA Main Source of HF peak





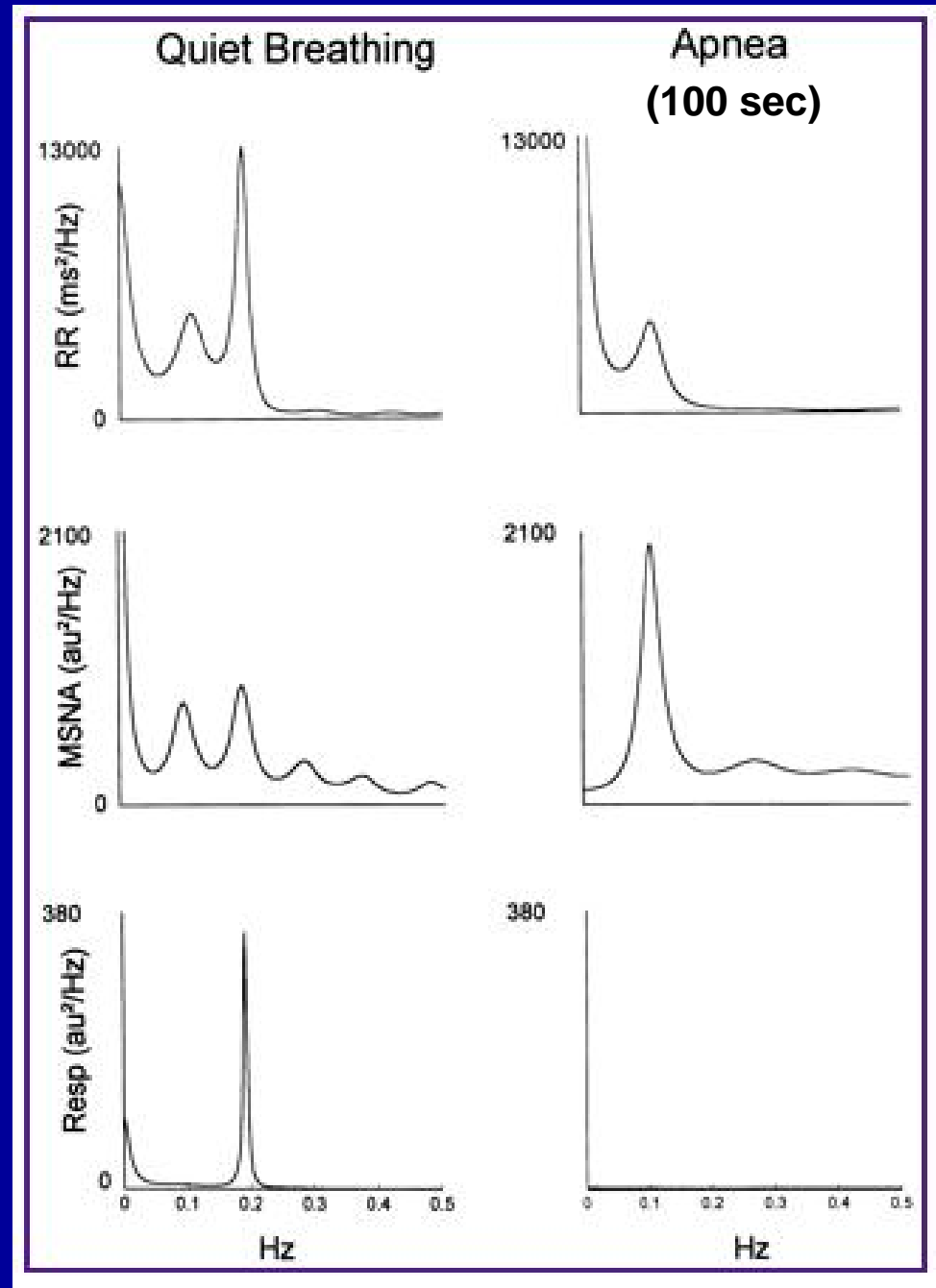
# Respiratory Linkage to HF & LF



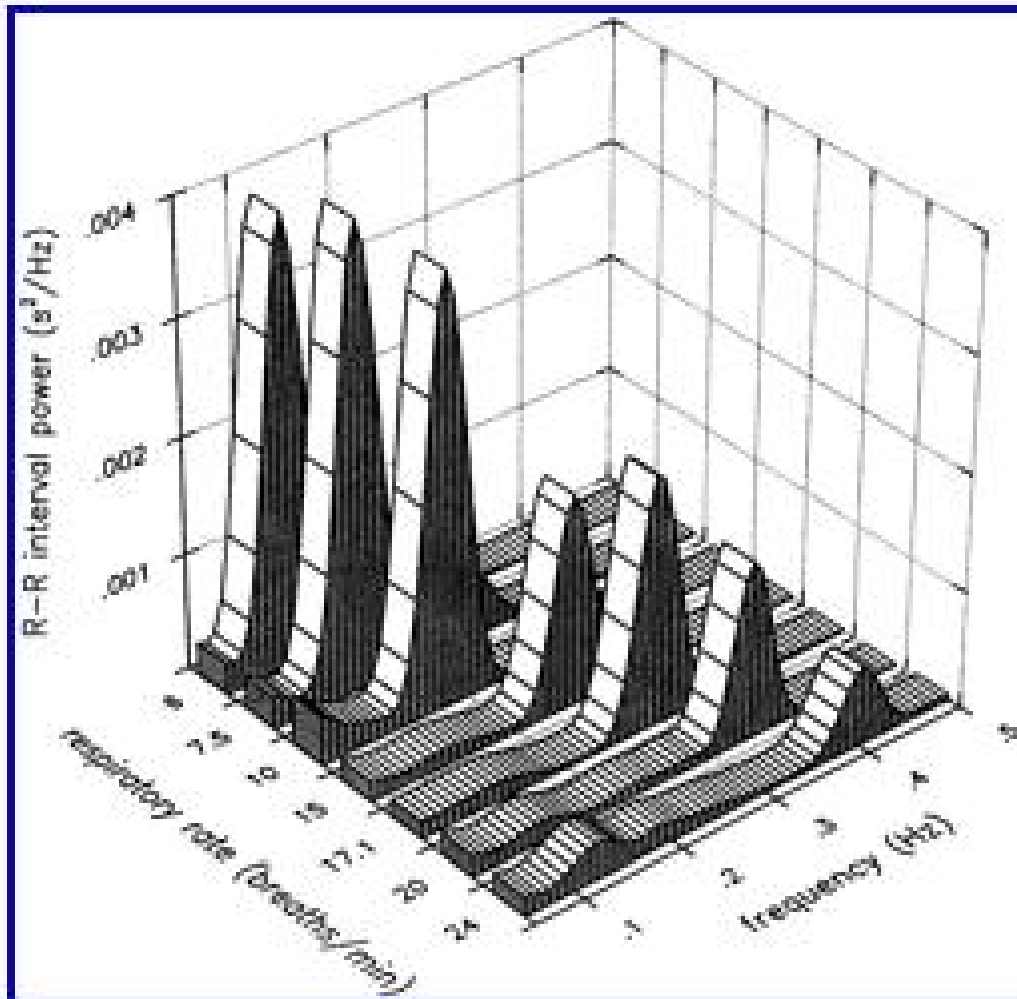
# Importance of Respiration

Peroneal nerve sympathetic

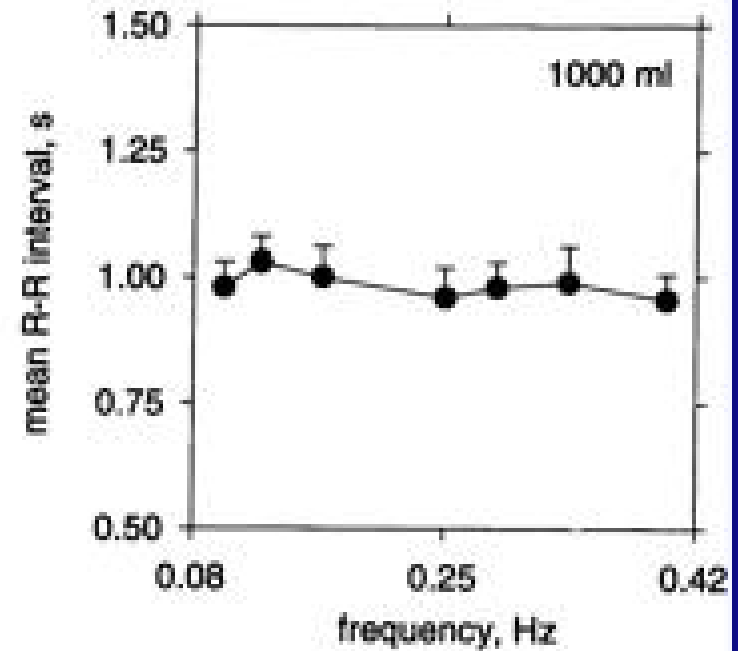
Paced Breathing  
At 0.2 Hz



# Slow rate allows fuller expression of Ach effects Resulting in greater HF power at lower frequencies



**Note HR itself  
DOES NOT CHANGE!**



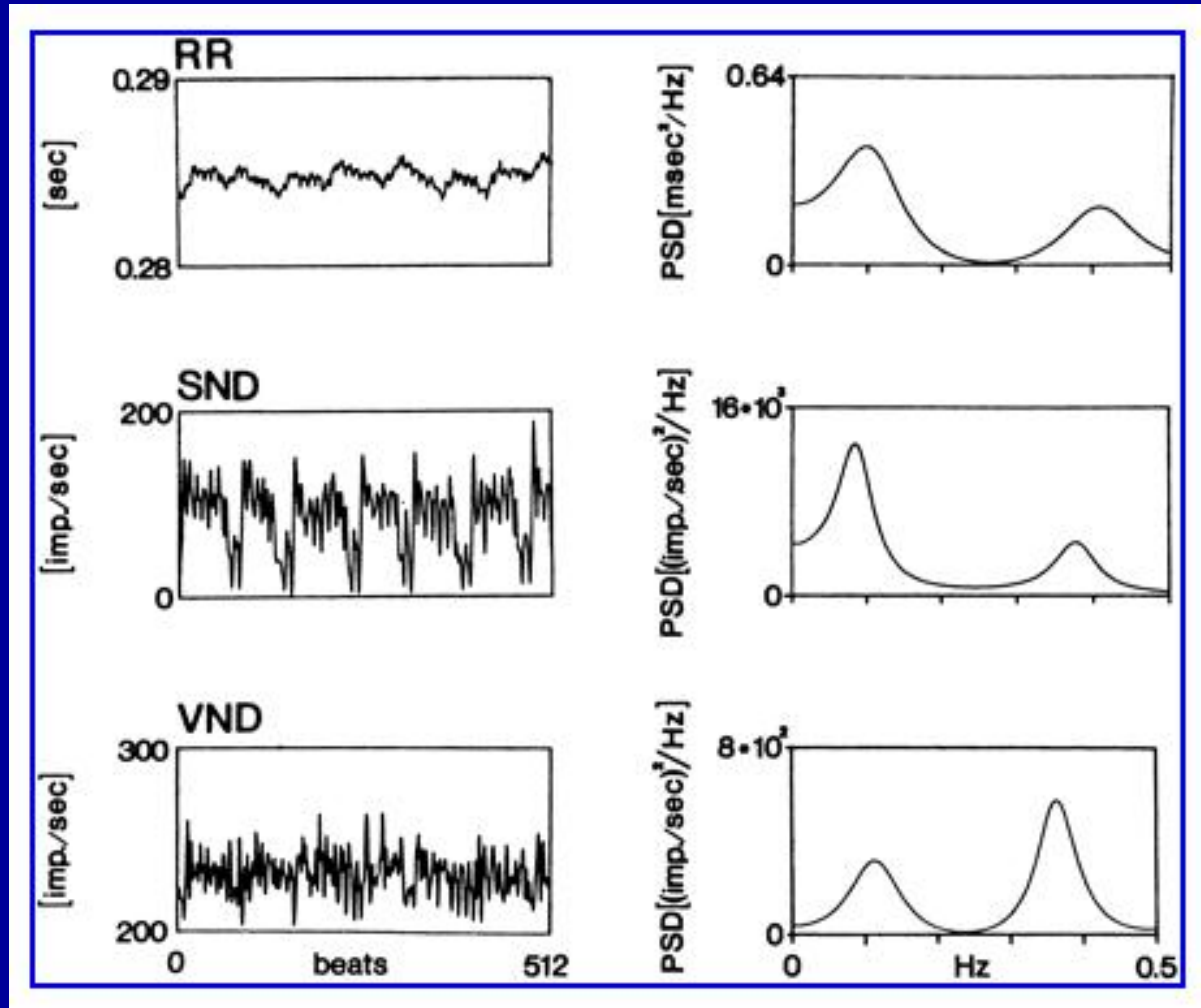
# Relationship to Neural Signals

R-R Interval

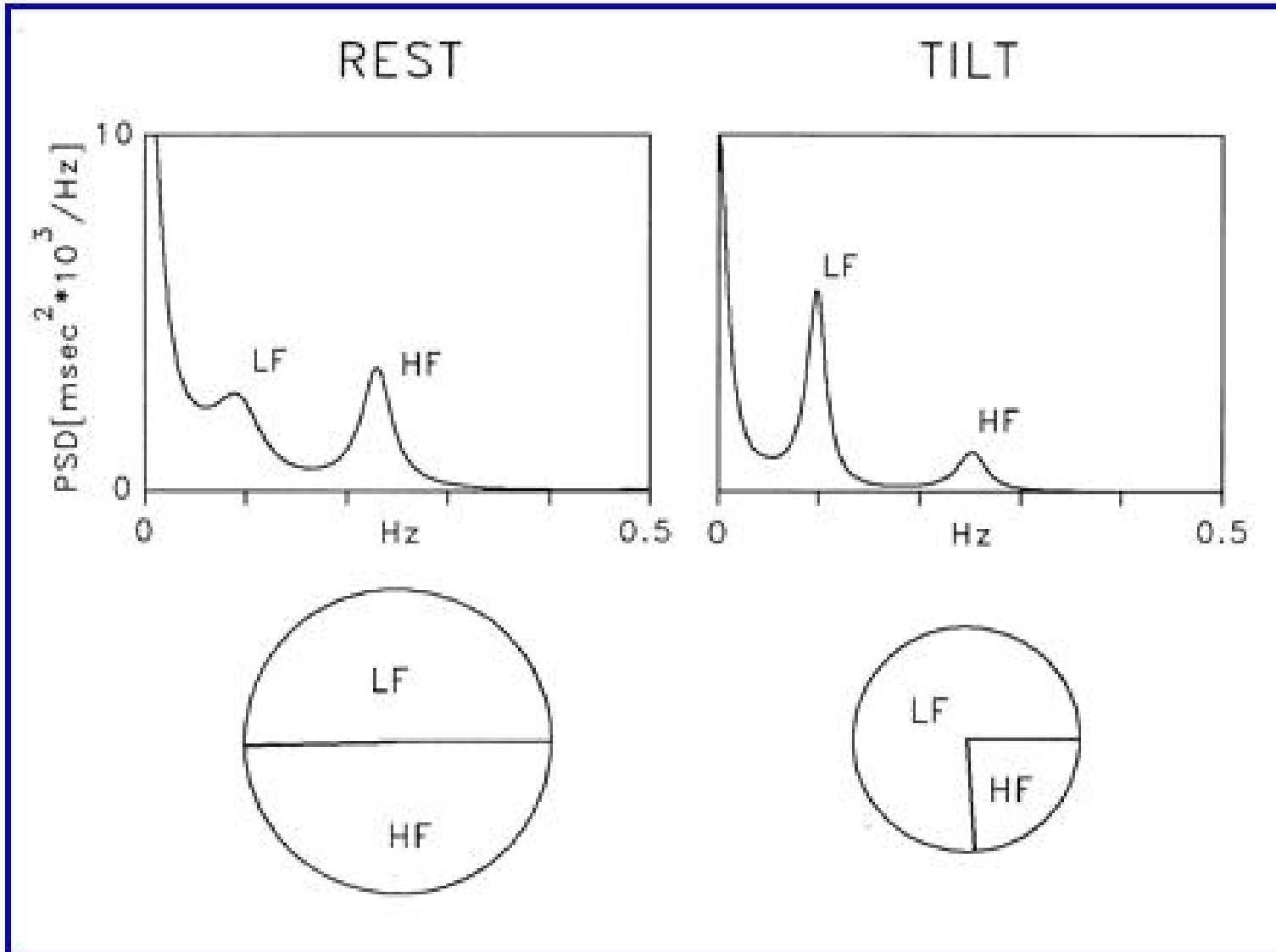
Cardiac Nerve Traffic

Sympathetic

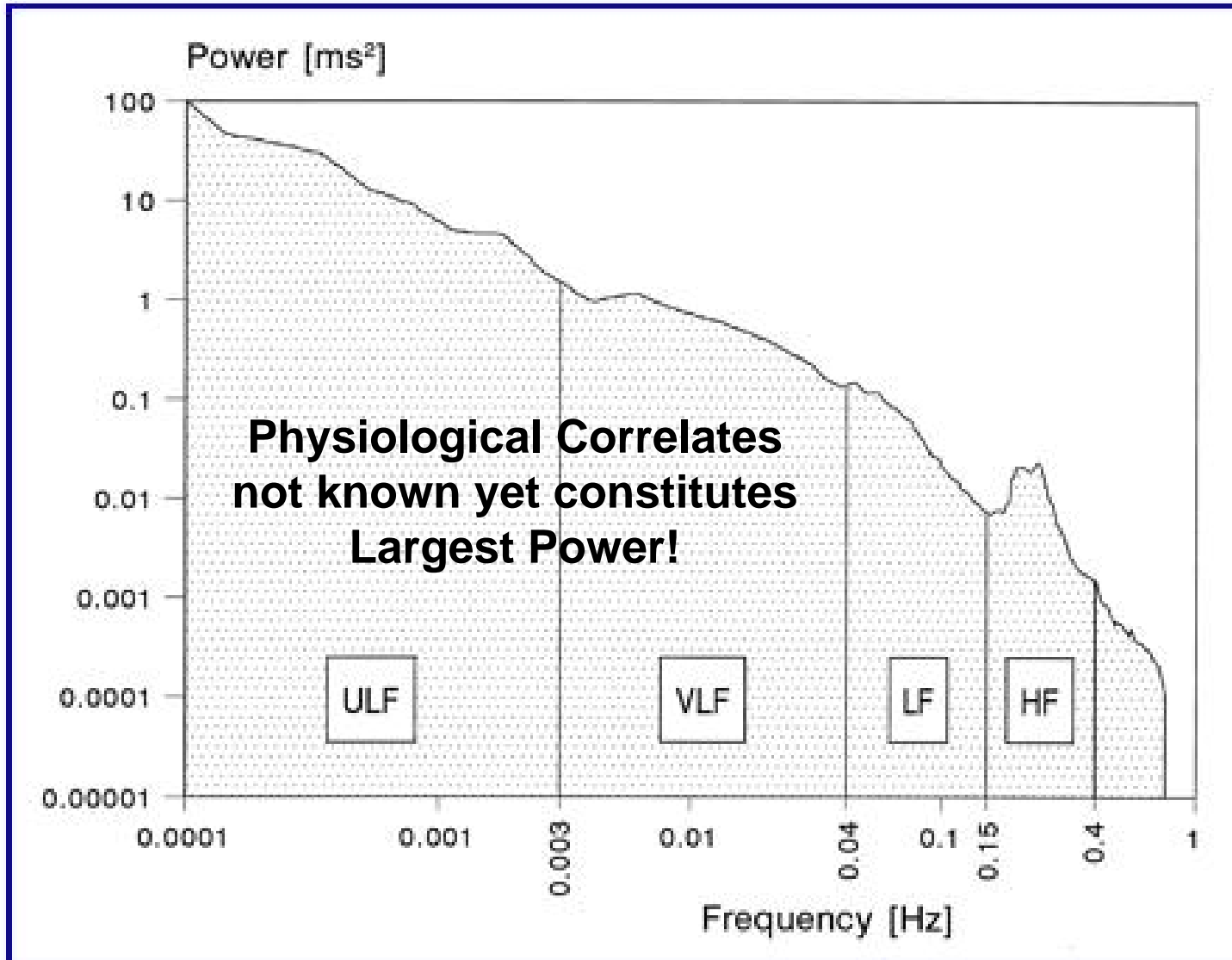
Vagal



# Enhancement of Sympathetic Modulation



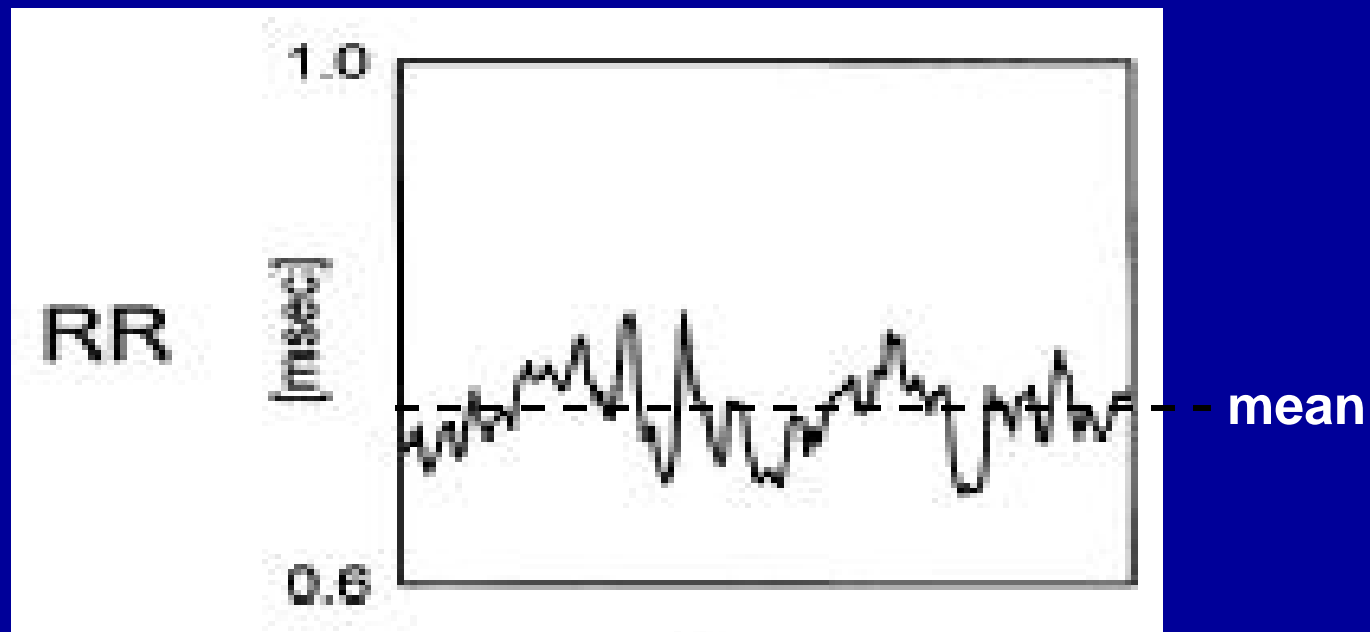
# 24 Hour Recording



# Time Analysis of HRV

uses standard deviation or variance  
of (normal) R-R intervals

$$\begin{aligned}\text{Coefficient of variance} &= \text{SD}/\text{mean} \\ &= \text{SDNN}/\text{mean}\end{aligned}$$

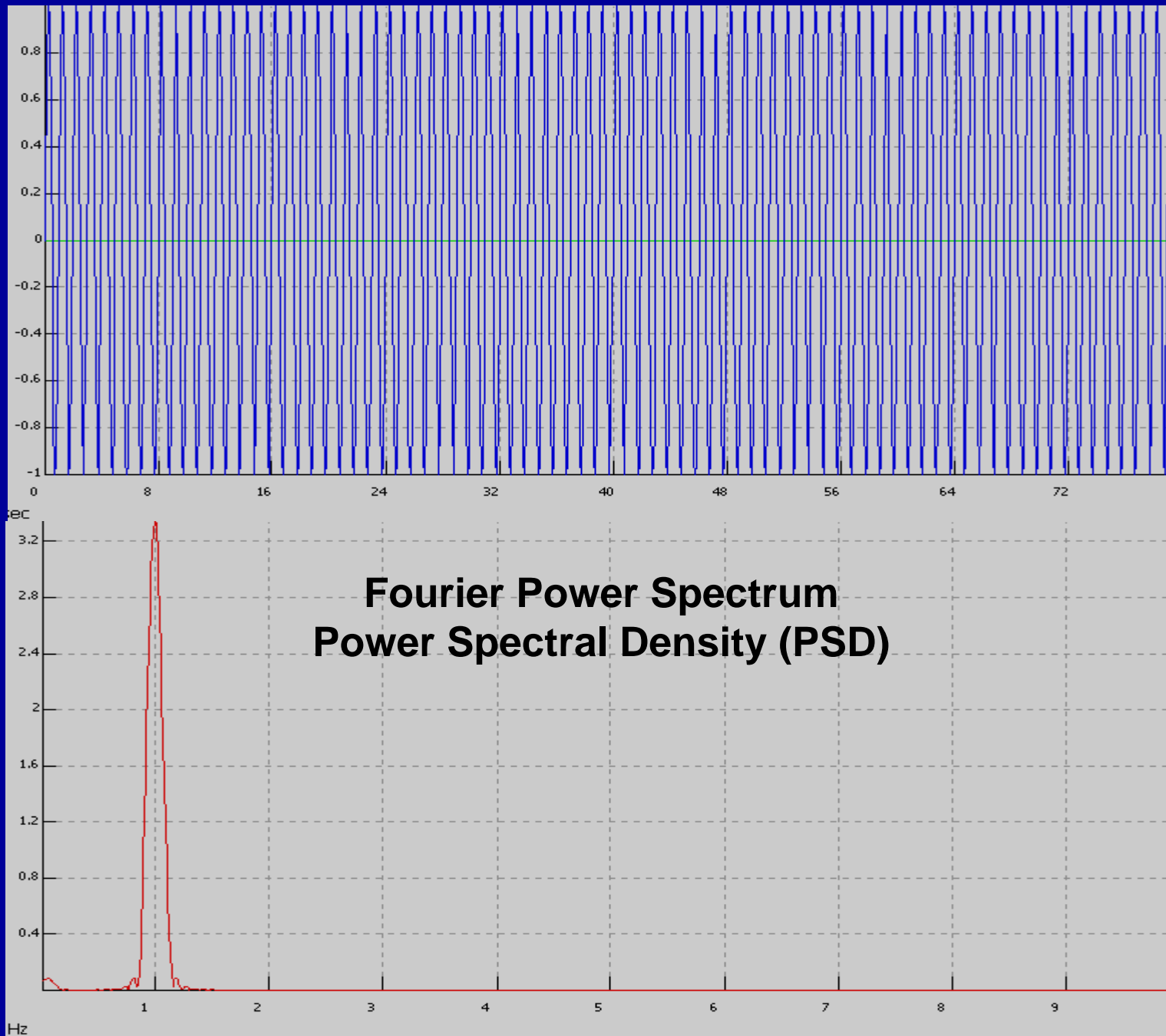


# **Spectral Analysis Considerations**

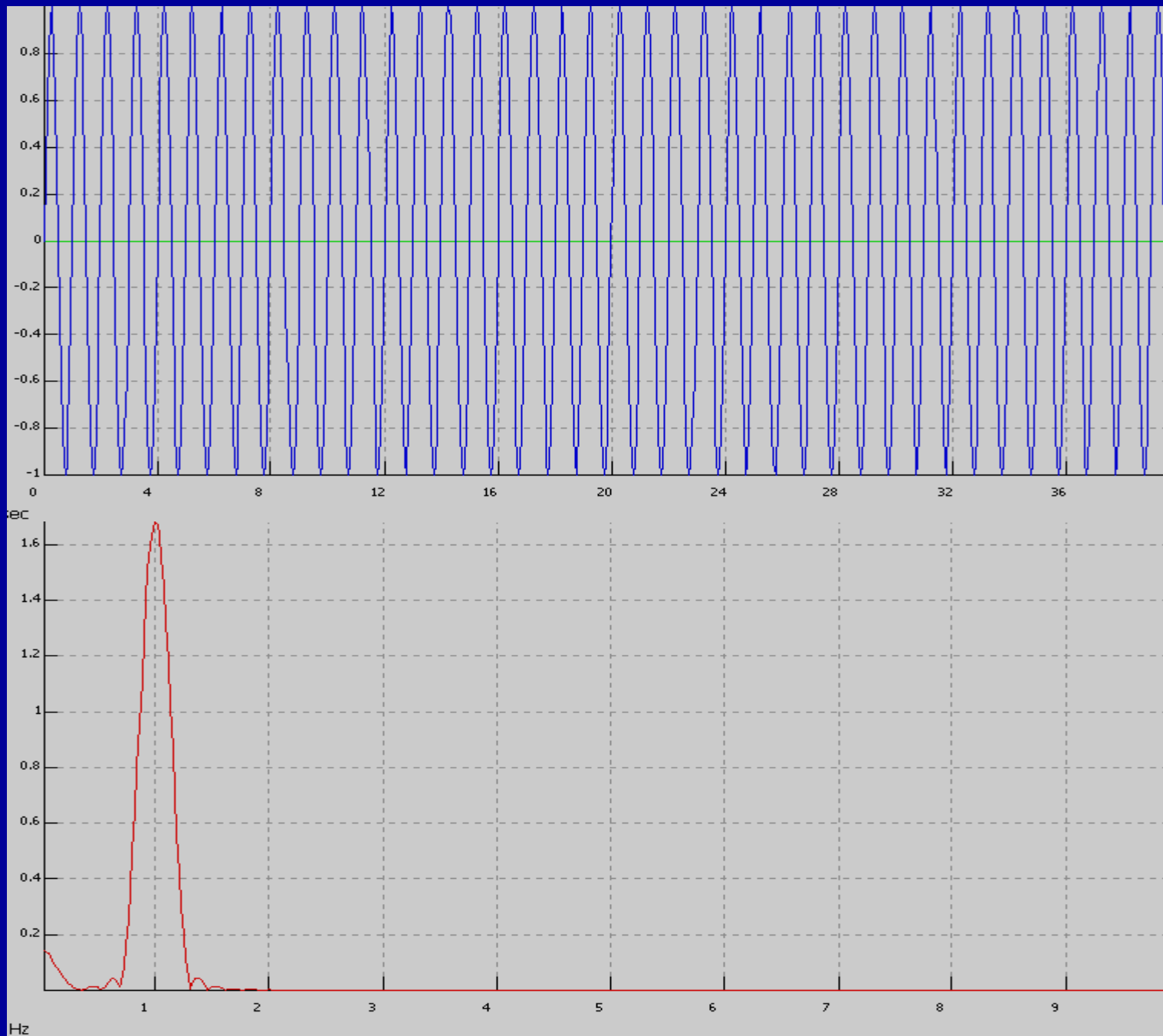


**For a given sampling rate the length  
of time a signal is sampled sets the  
Frequency Resolution**

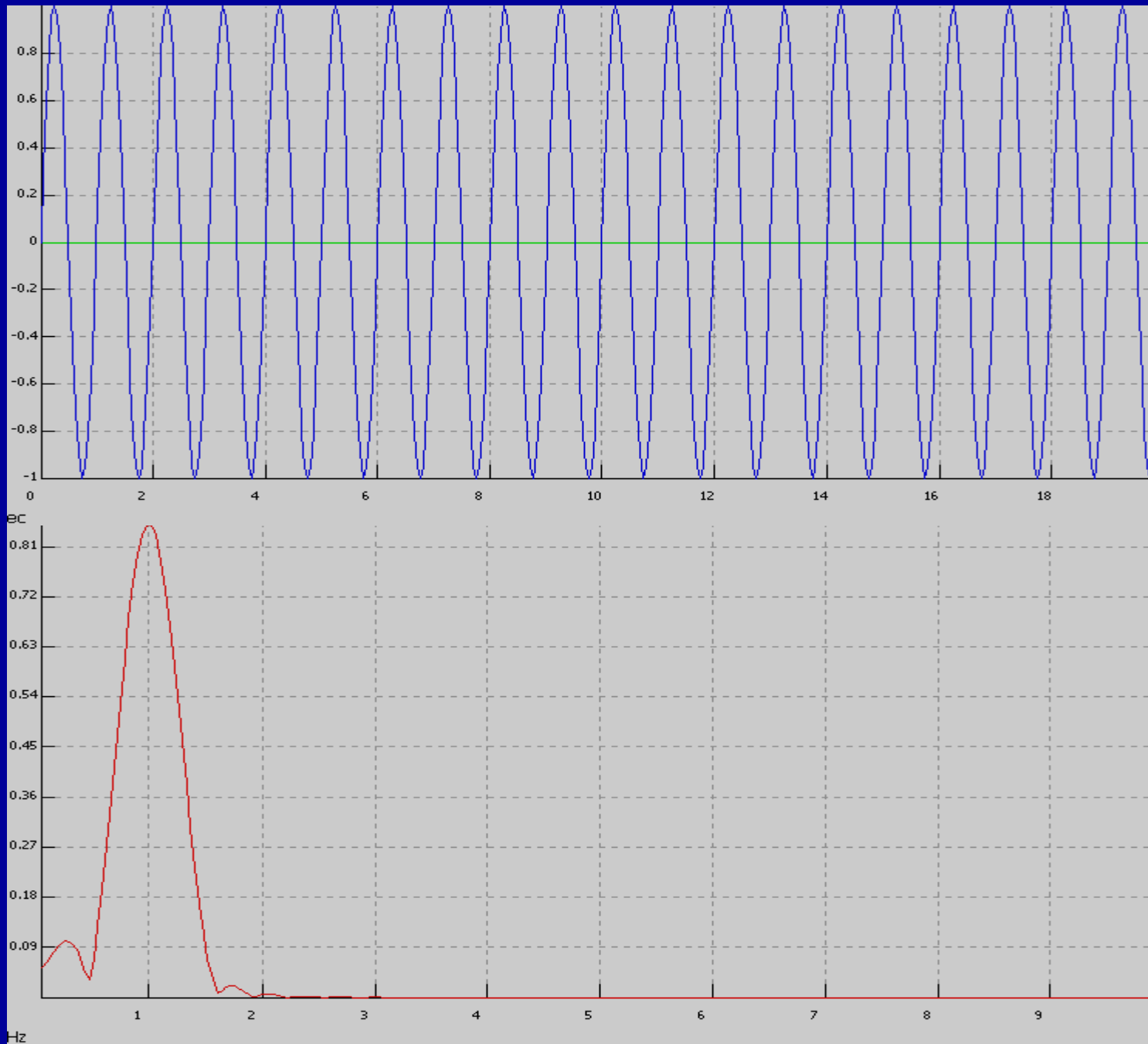
Signal  
80 cycles  
of a 1 Hz  
sine  
wave



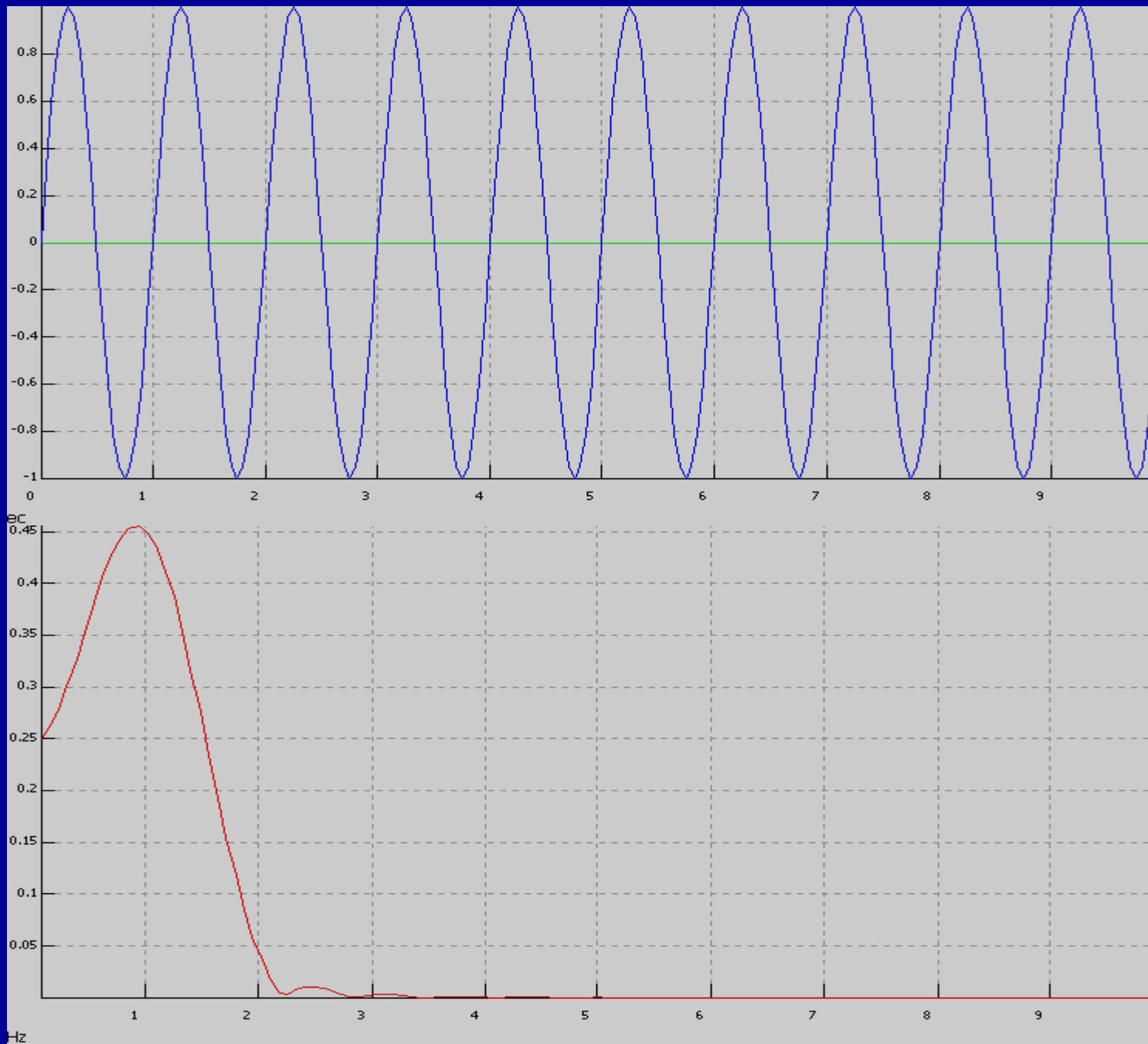
Signal  
40 cycles  
of a 1 Hz  
sine  
wave



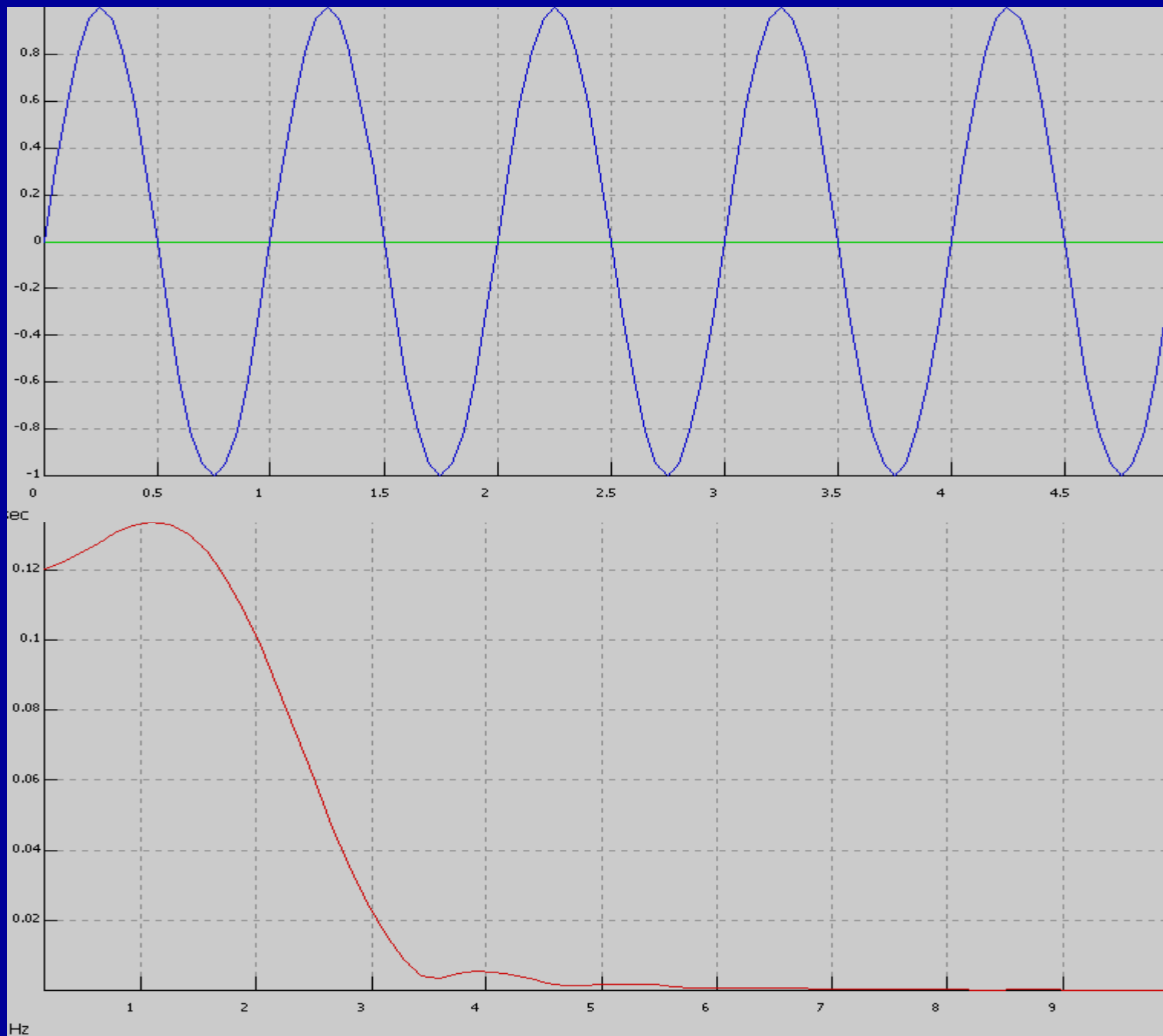
Signal  
20 cycles  
of a 1 Hz  
sine  
wave



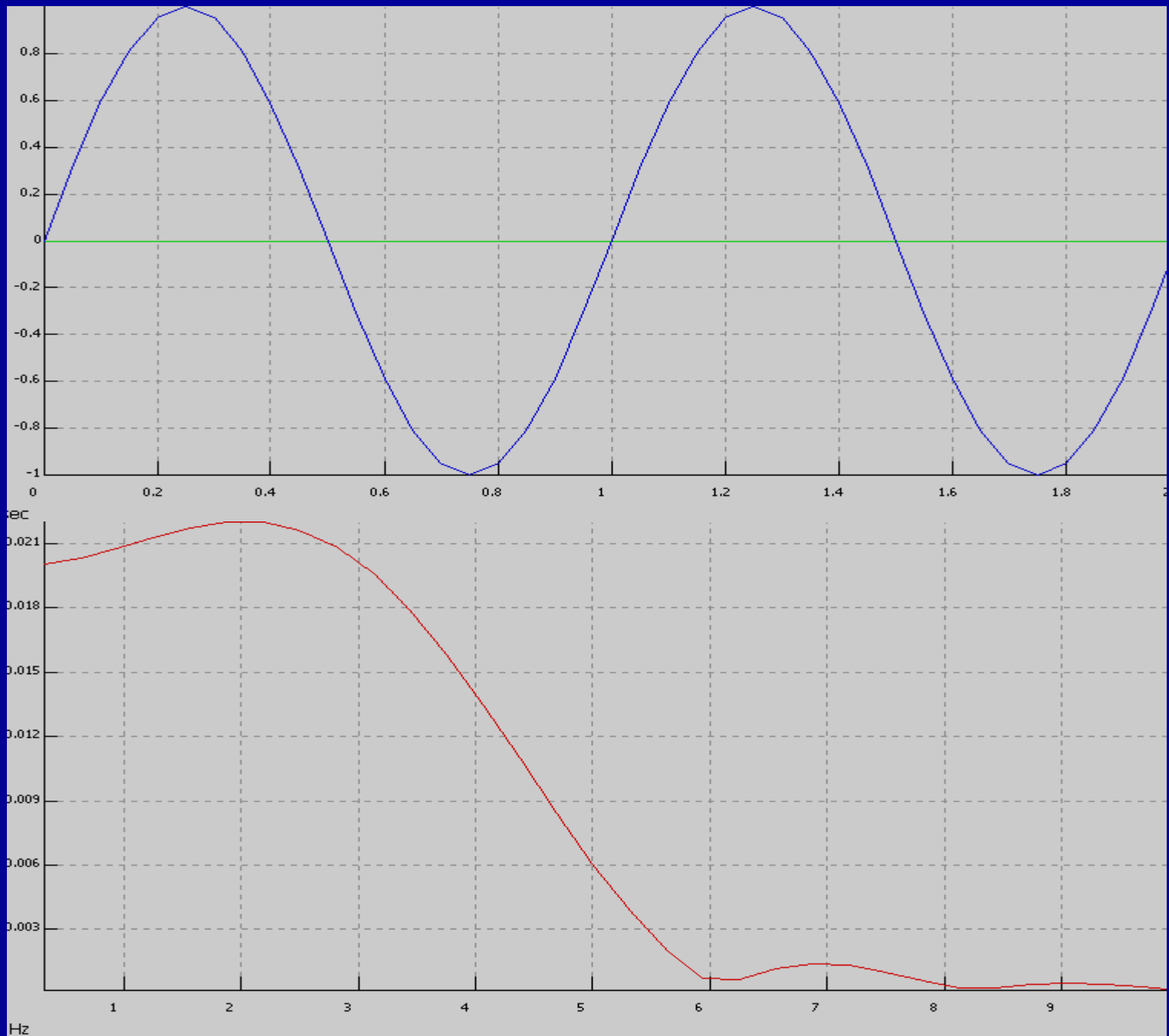
**Signal**  
**10 cycles**  
**of a 1 Hz**  
**sine**  
**wave**



**Signal**  
**5 cycles**  
**of a 1 Hz**  
**sine**  
**wave**



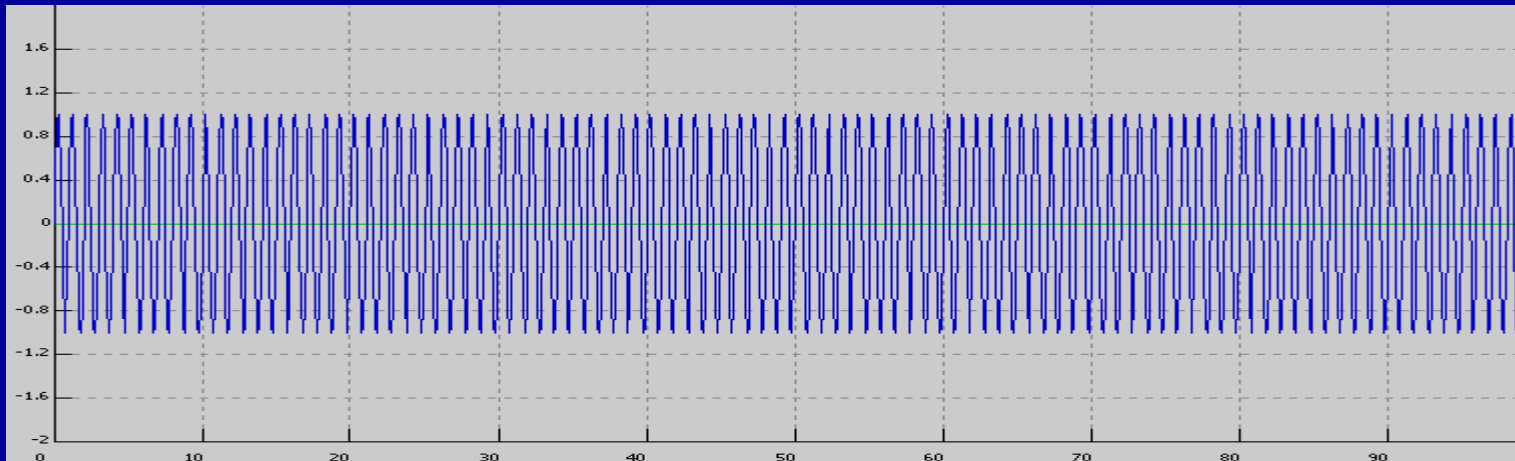
**Signal**  
**2 cycles**  
**of a 1 Hz**  
**sine**  
**wave**



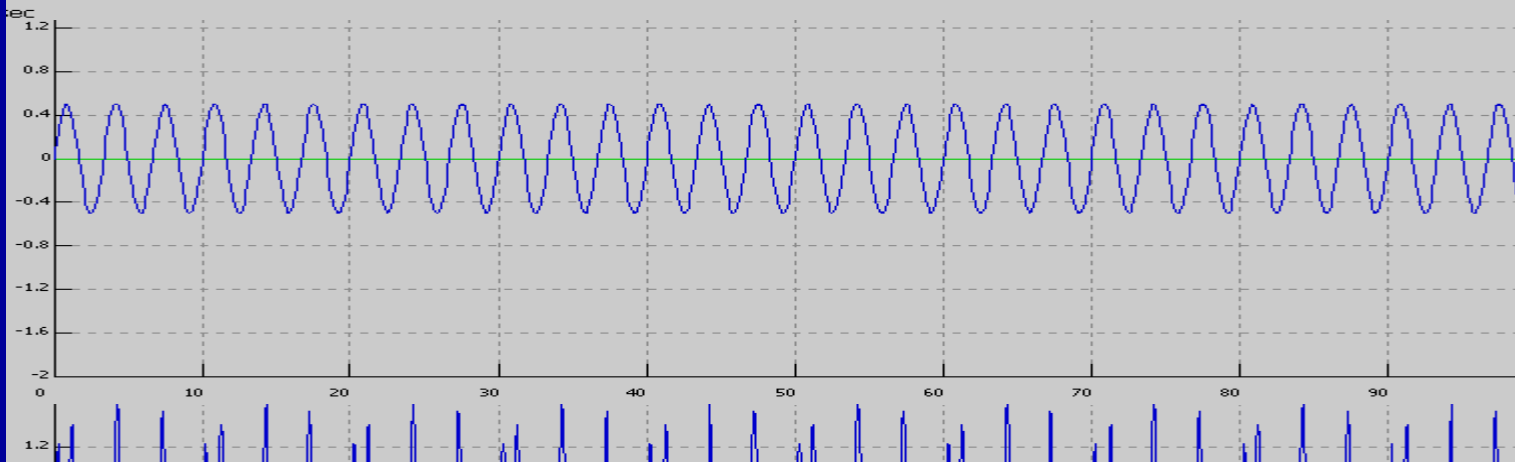
**Seperating frequency components  
requires adequate resolution**



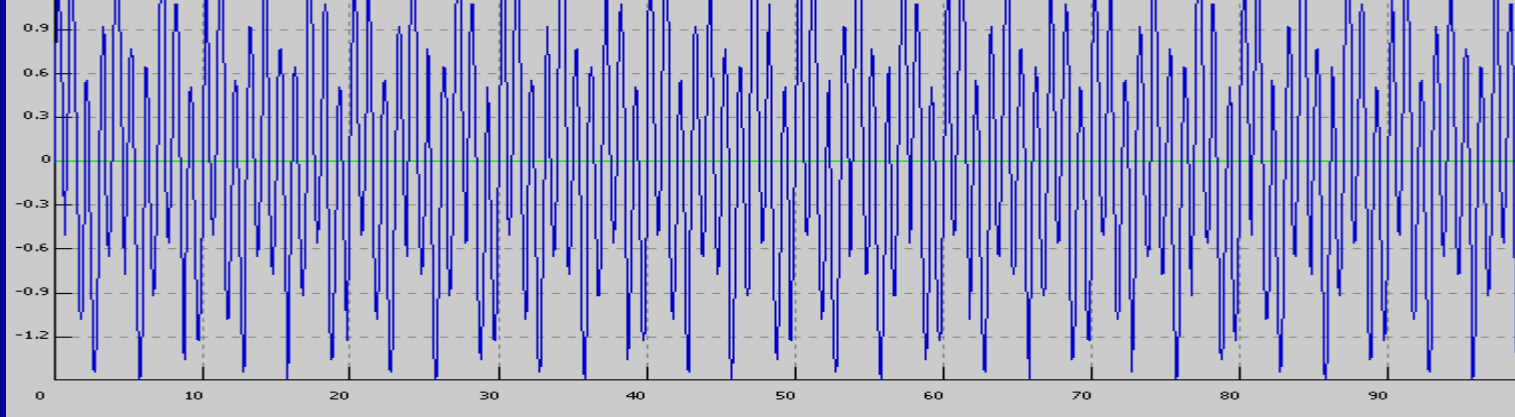
**A**  
1.0 Hz



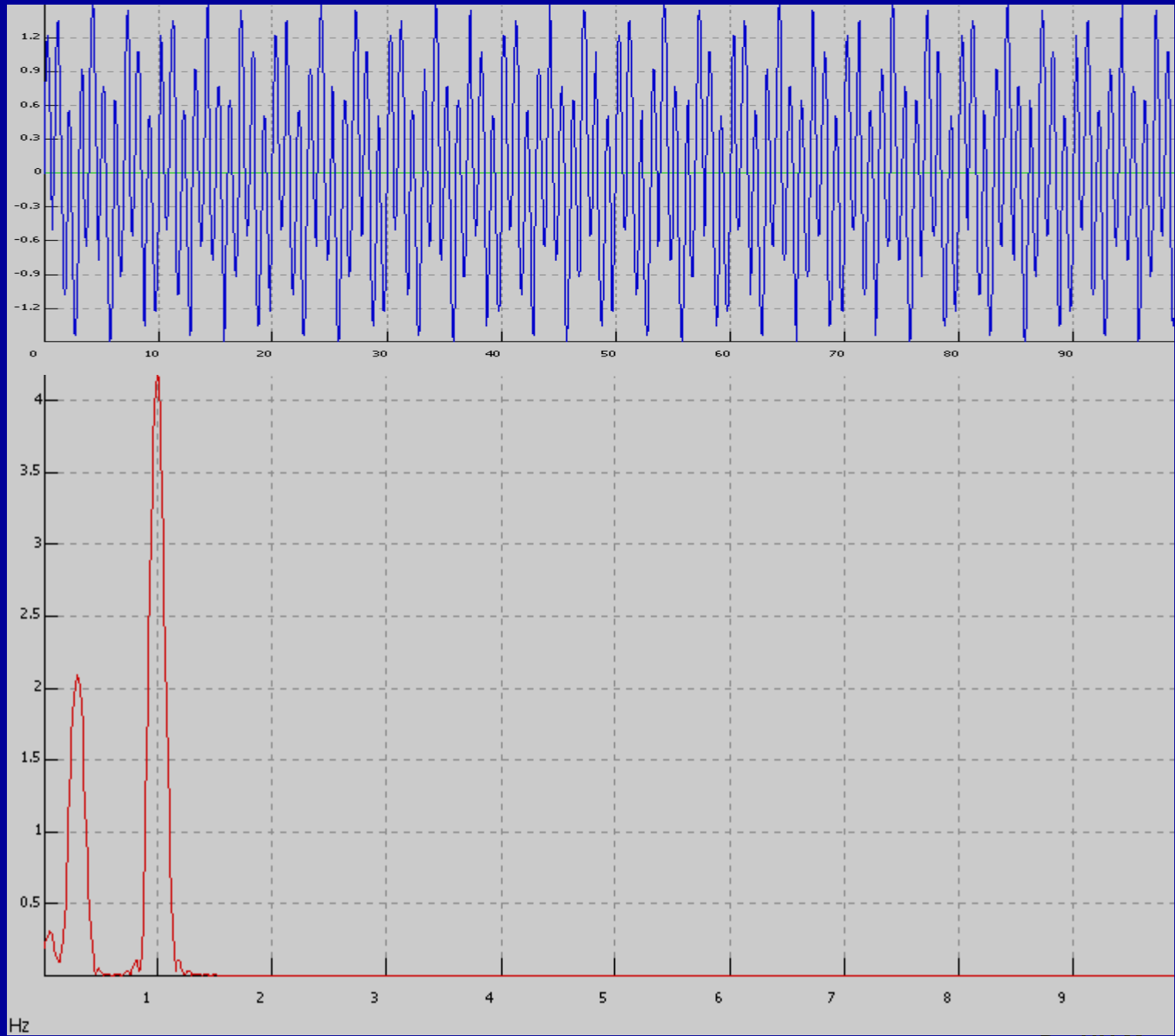
**B**  
0.3 Hz



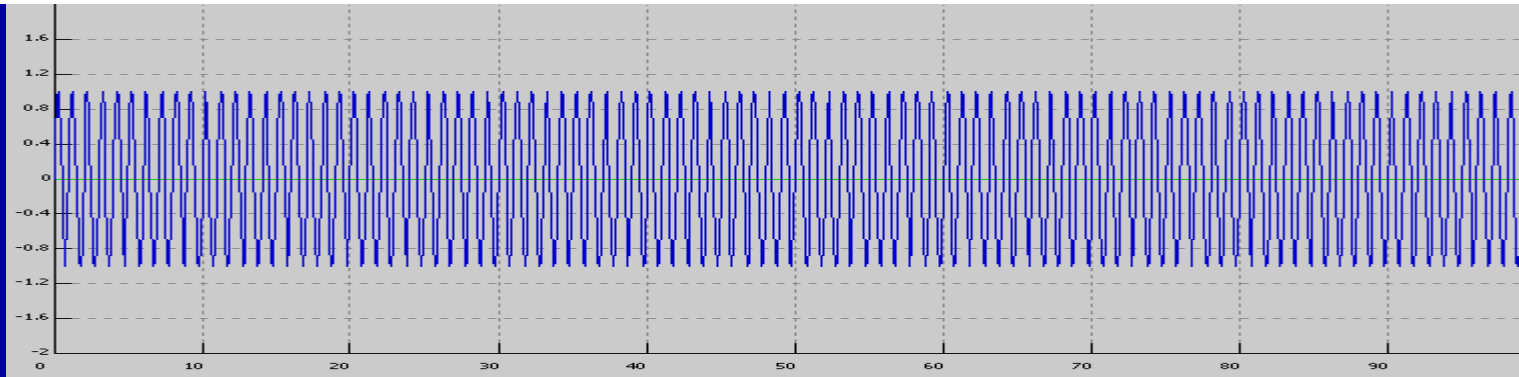
**A + B**



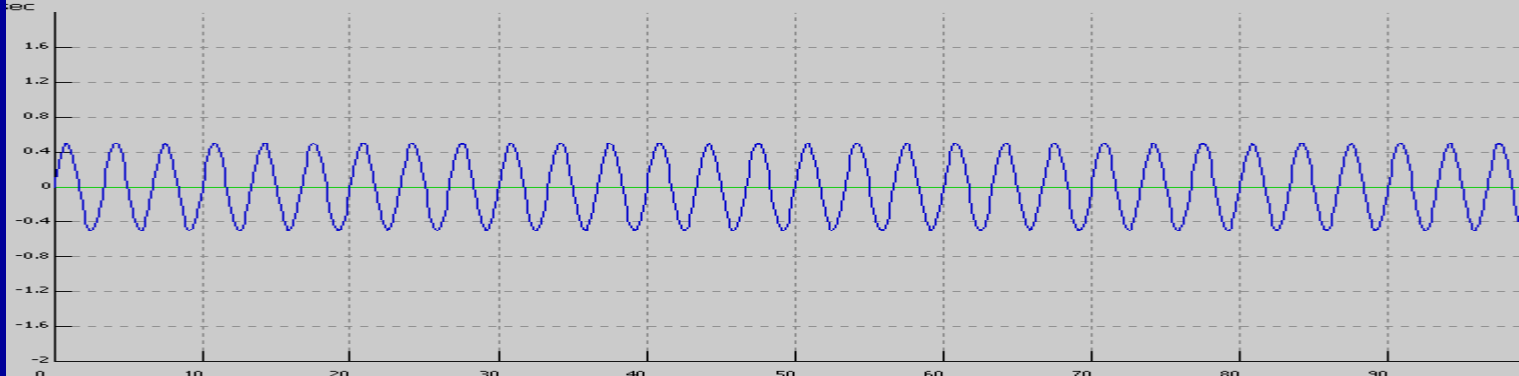
**A + B**



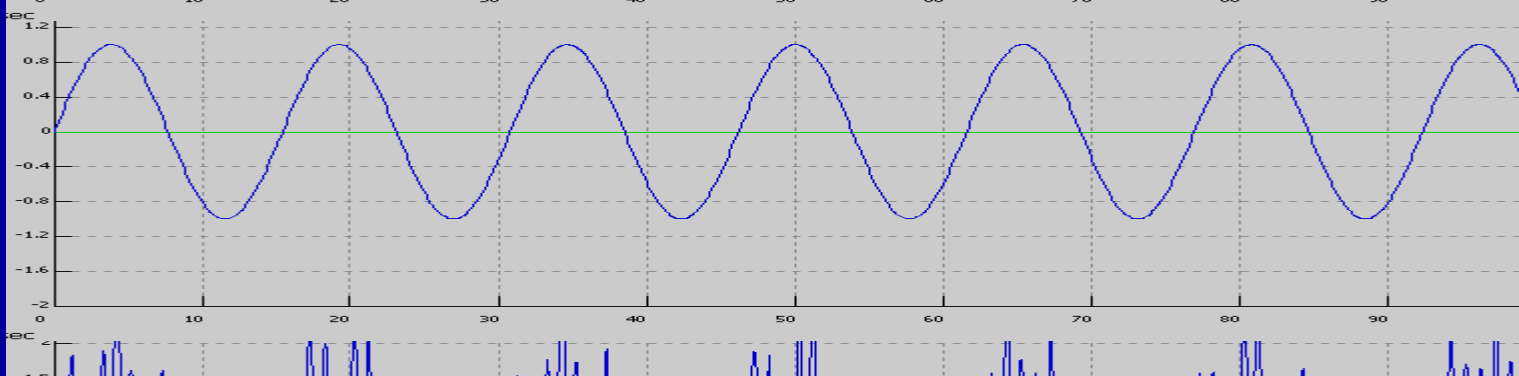
**A**  
1.0 Hz



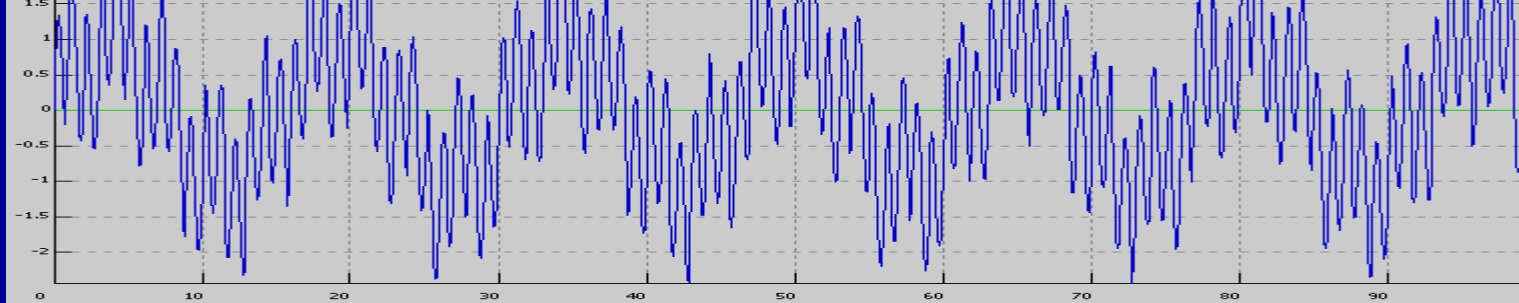
**B**  
0.3 Hz



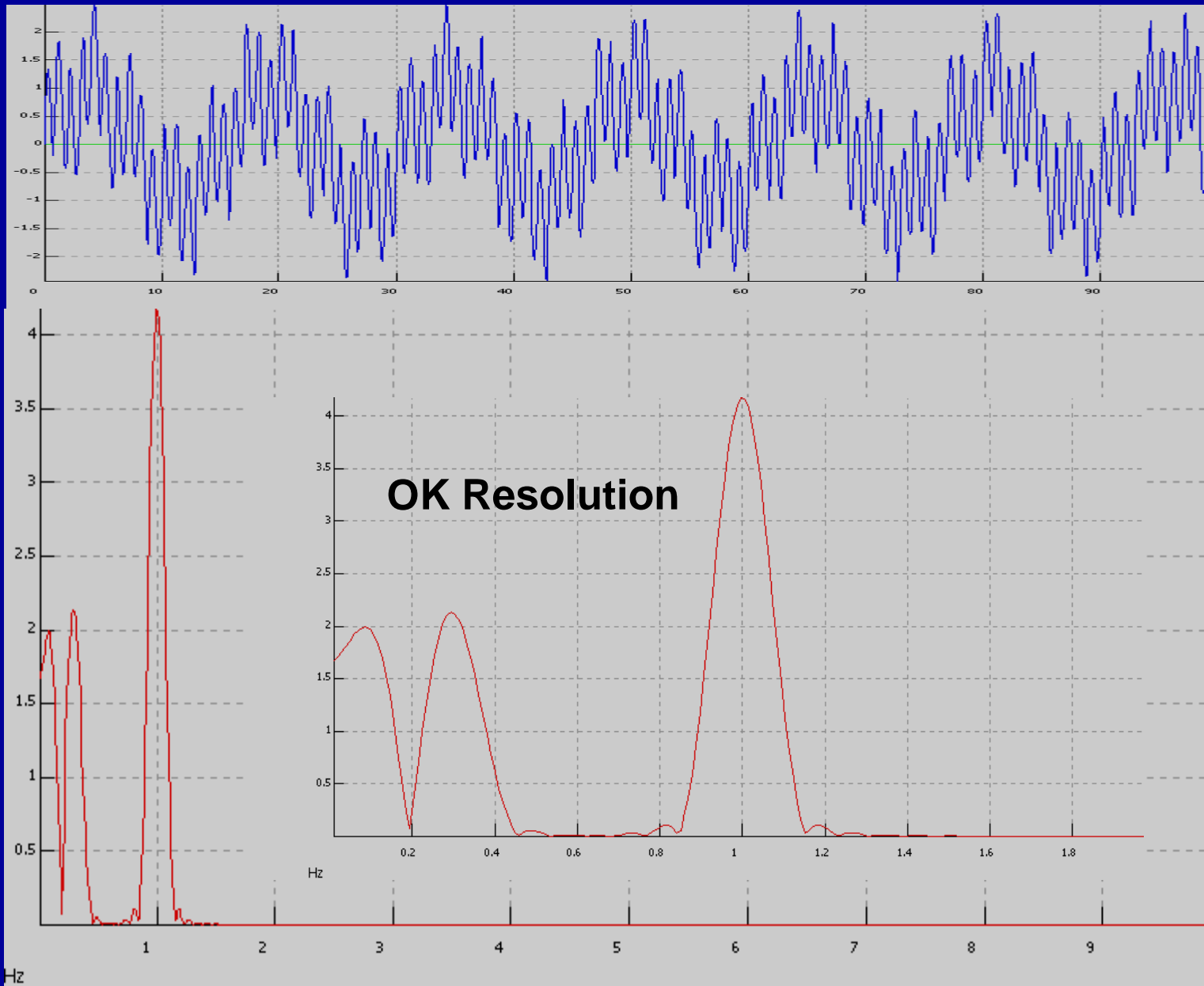
**C**  
0.065 Hz



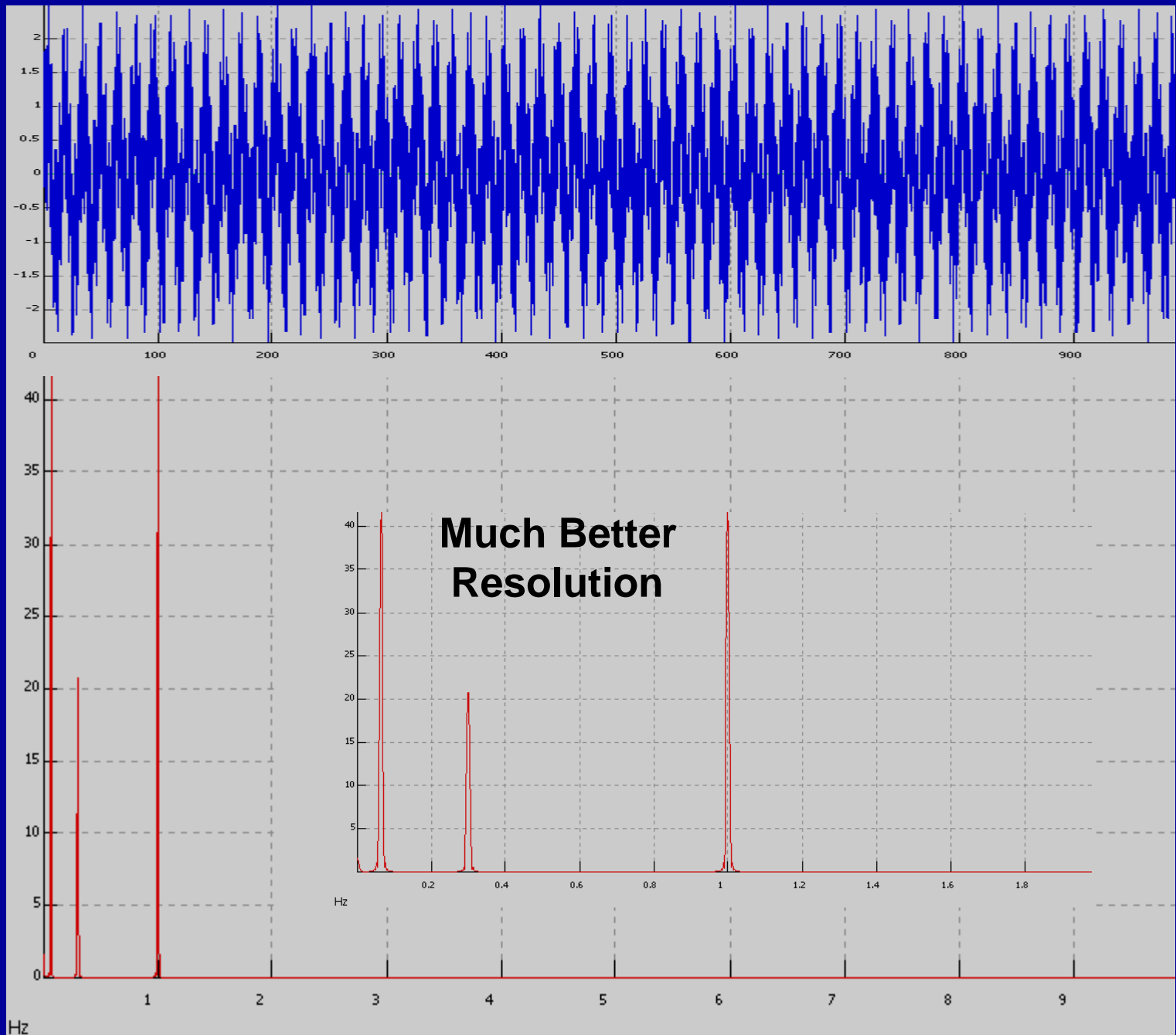
**A+ B+ C**

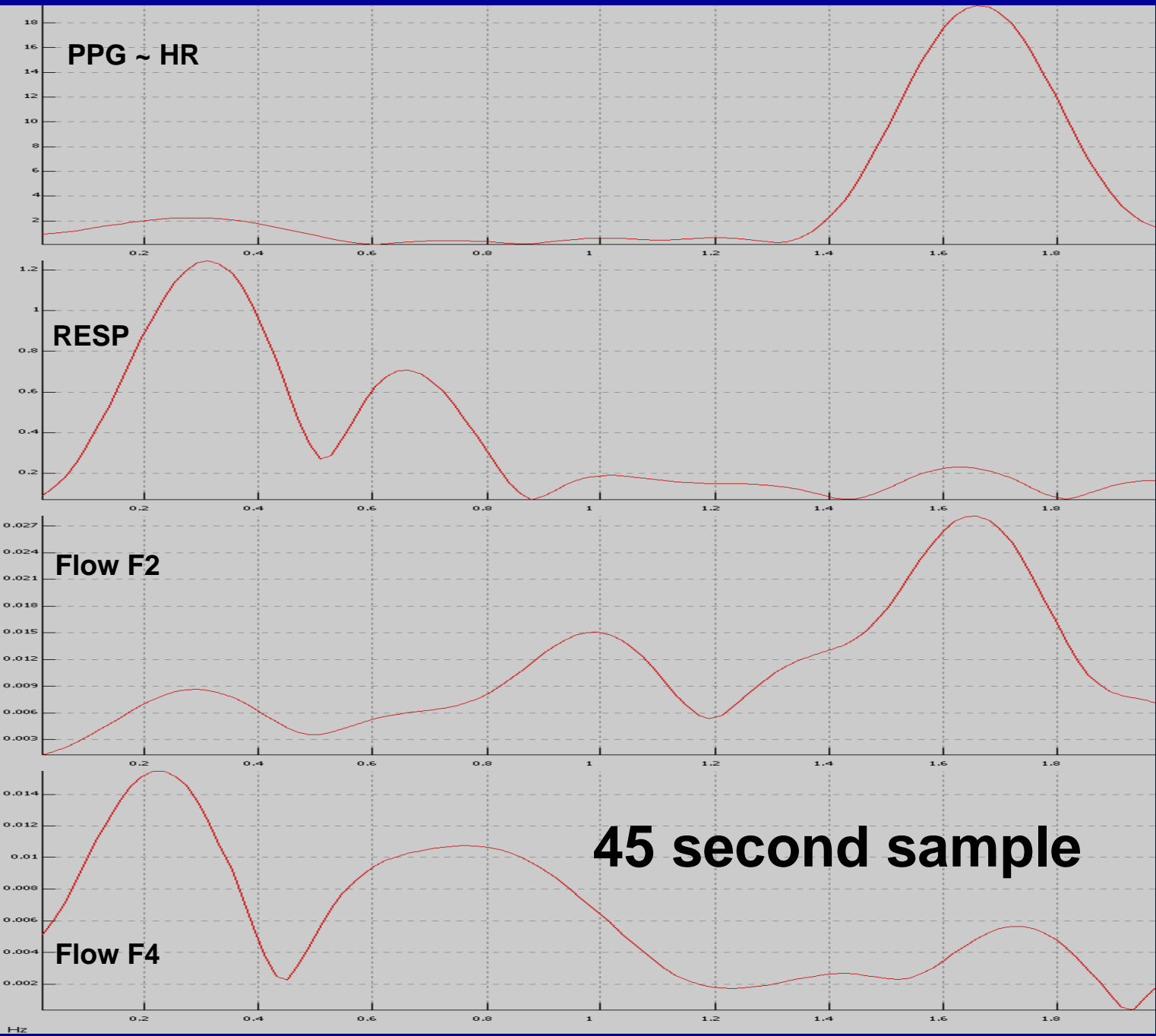


**A+ B+ C**  
**100 sec**



1000 sec





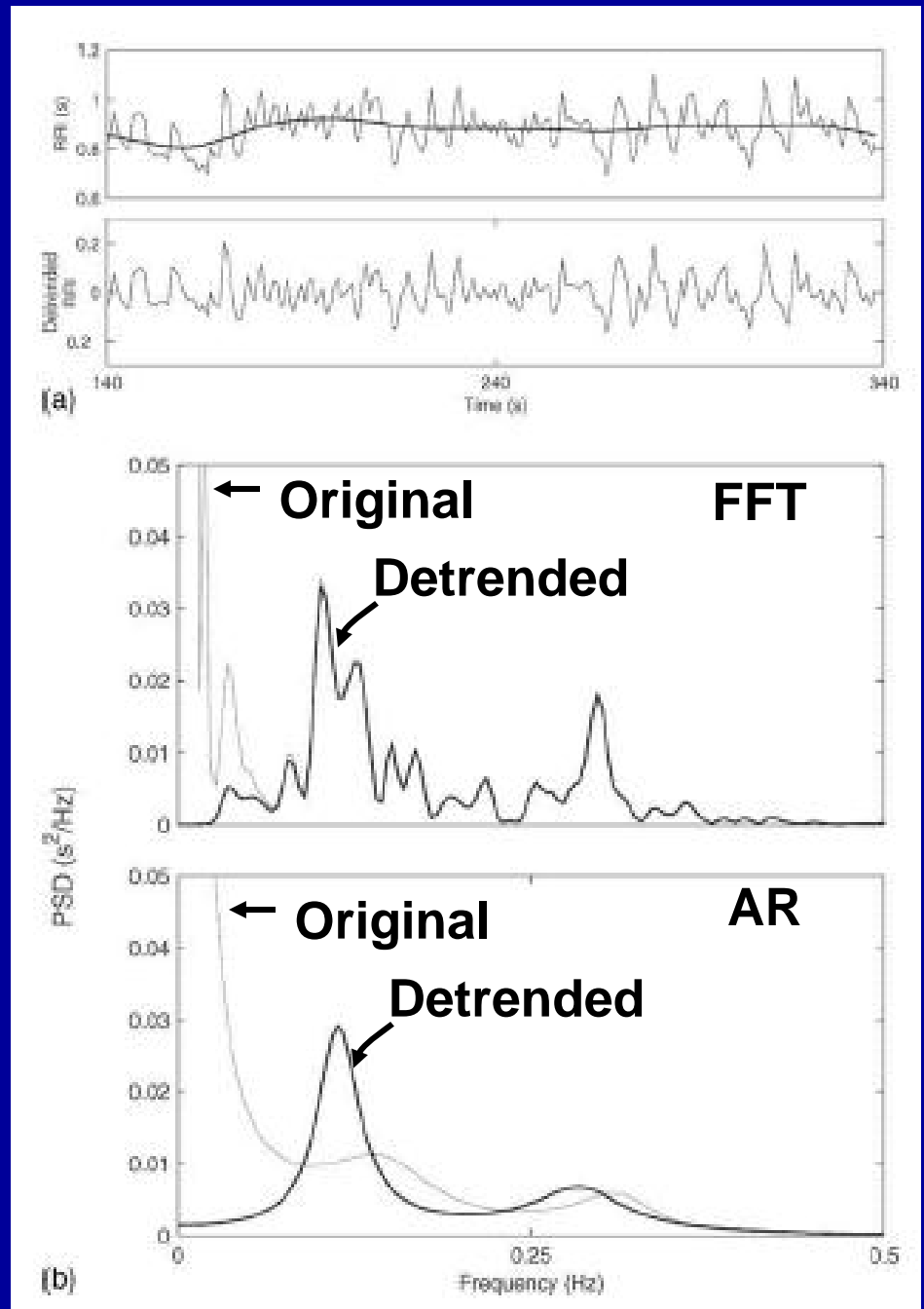
**45 second sample**

# De-Trending

R-R Interval series as obtained

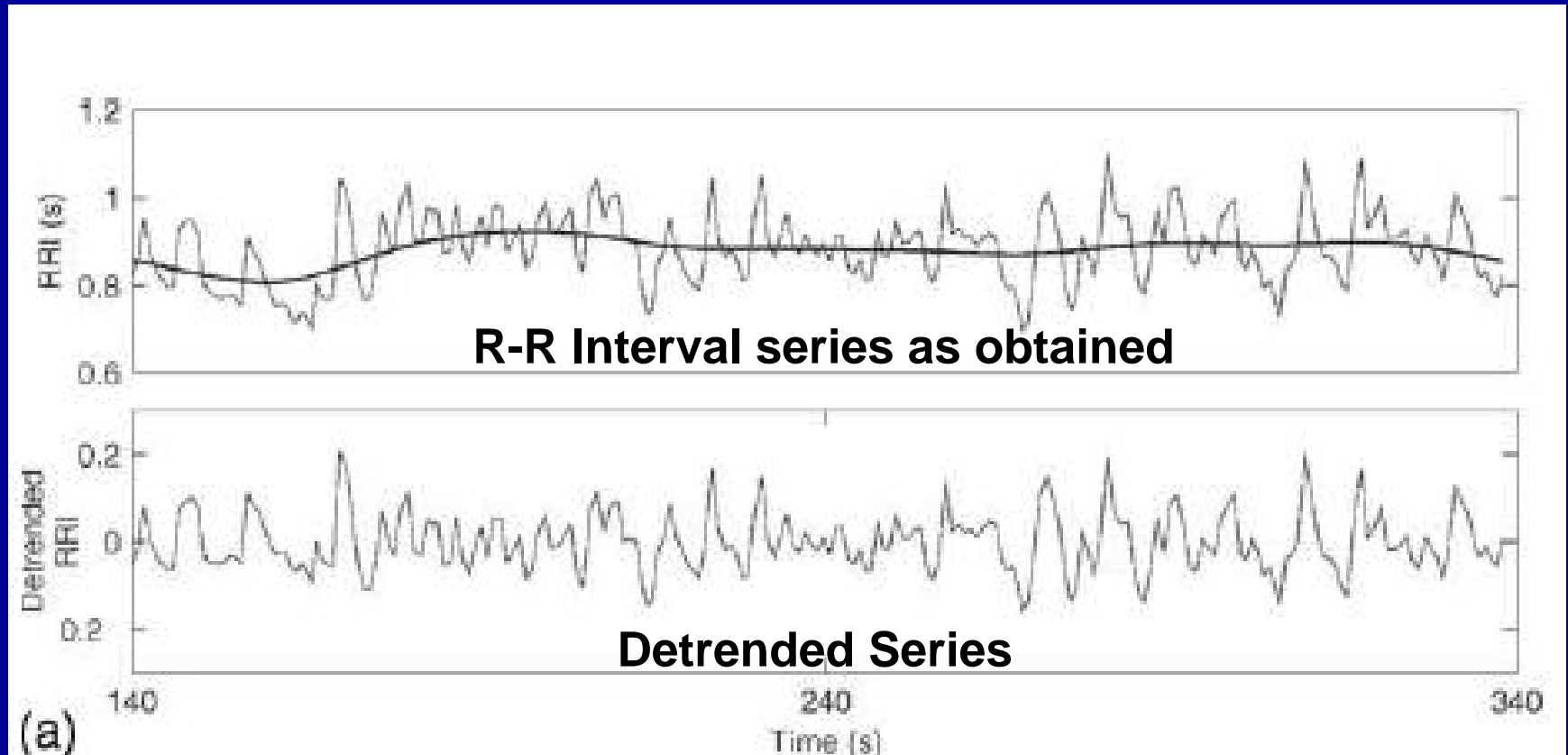
Detrended Series

Power Spectral Density  
(PSD) of R-R Series





# Effect of Detrending

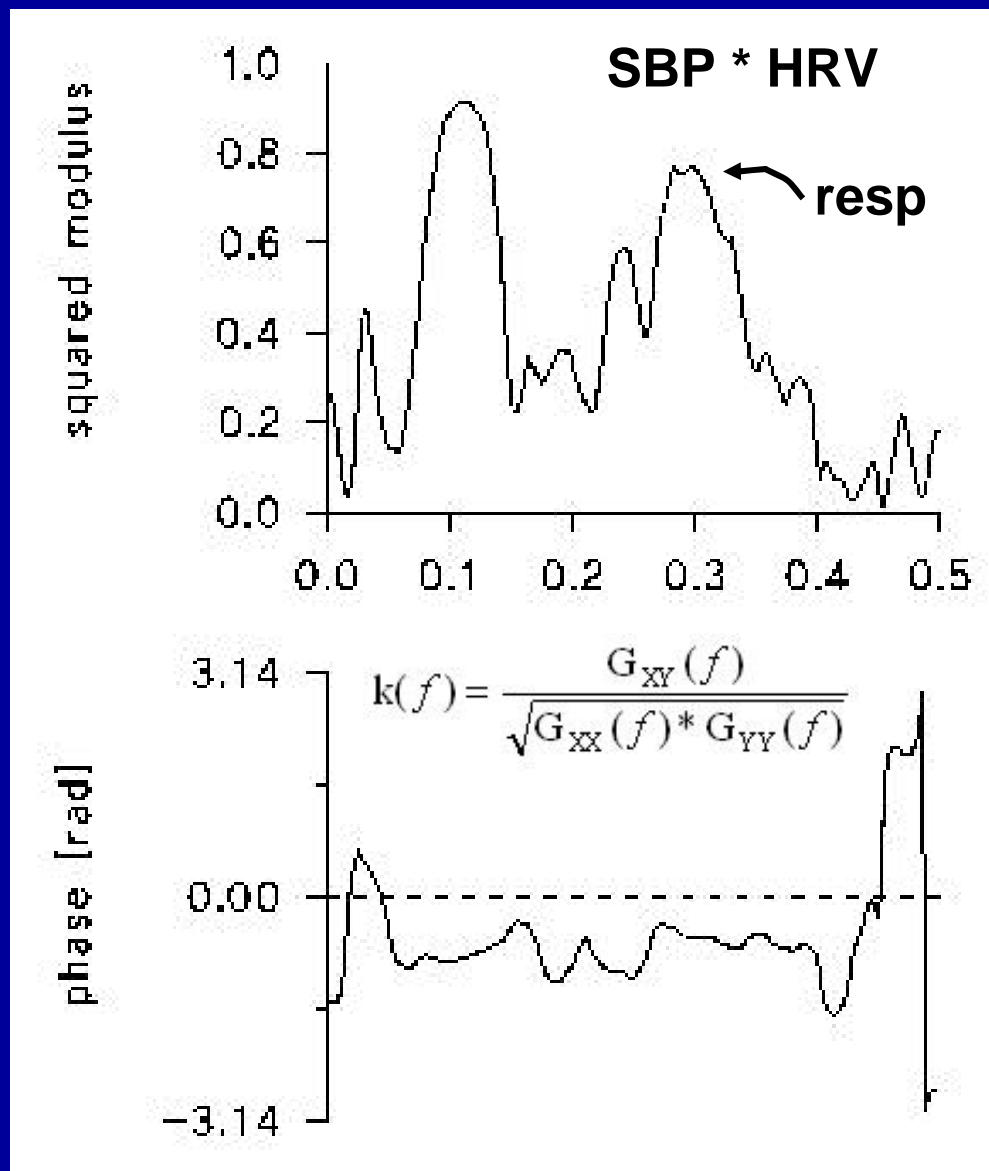


# Basic Definitions

# Coherence Function - Degree of linear correlation as fn of frequency

$G_{xx}$ ,  $G_{yy}$  and  $G_{xy}$  are spectra of  $x(t)$ ,  $y(t)$  and crossspectrum of  $x$  and  $y$

$$[K(f)]^2$$



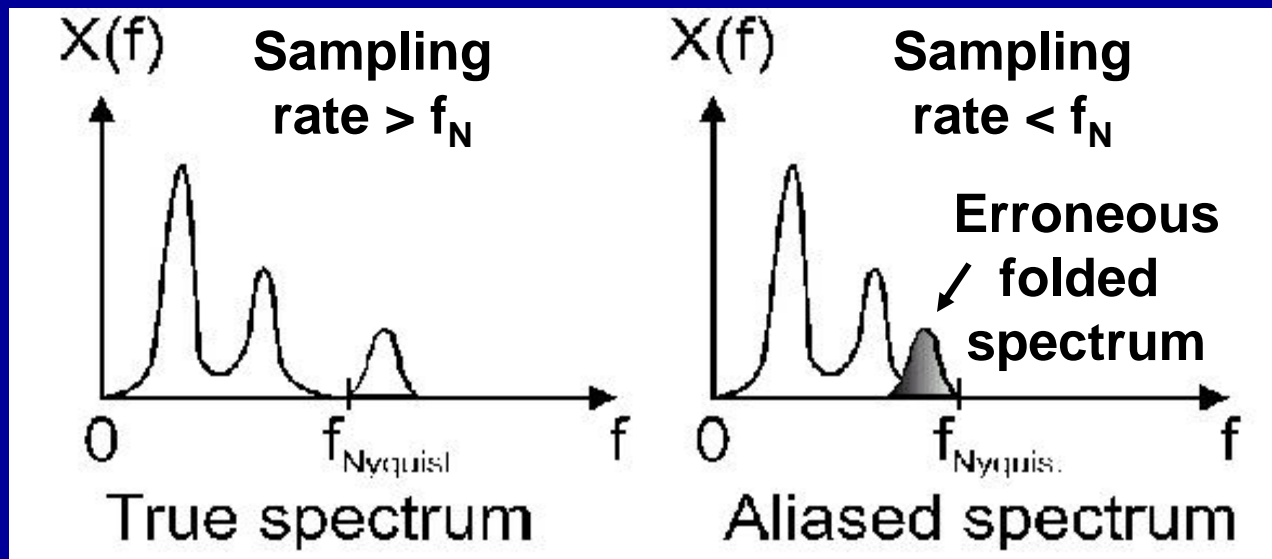
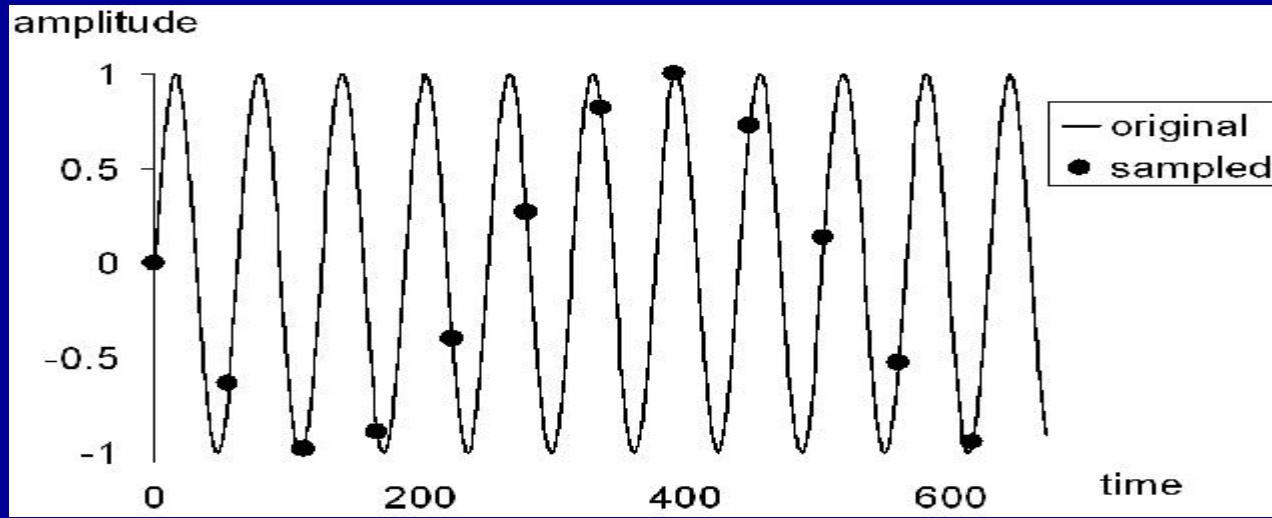
If  $x(k)$  is the  $k$ -th value of a time series of  $N$  samples with sampling period  $\Delta t$ , its energy  $E$  is defined as:

$$E = \sum_{k=0}^{N-1} |x(k)|^2 \Delta t$$

$$P = \frac{E}{N\Delta t} = \frac{1}{N} \sum_{k=0}^{N-1} |x(k)|^2$$

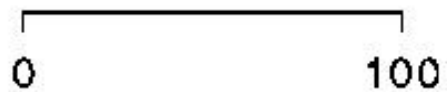
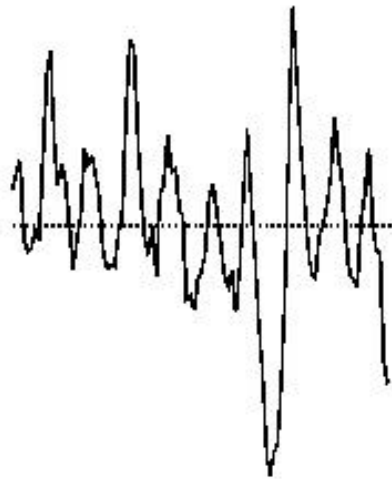
For zero-mean time series, the power is equal to the variance of the sample of the  $N$  values  $x(k)$ .

# Aliasing Artifacts



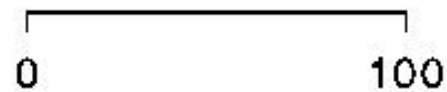
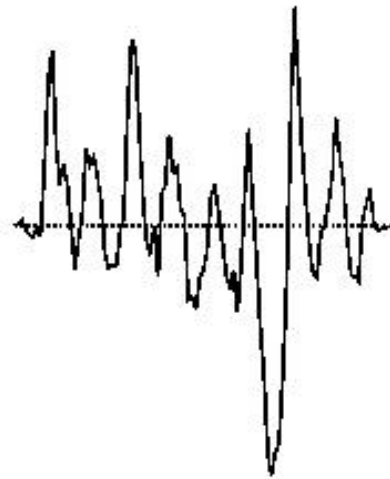
# Windowing

no windowing

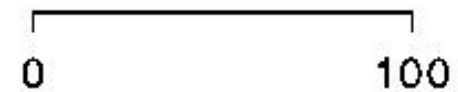


$w(t) =$

10% cosine-taper



Hann



# Autocorrelation Function

Measure of the dependence of time series values at one time on the values at another time.

Given the time series  $x(n)$ ,  $n=1, 2, \dots, N$ , the autocorrelation function at lag  $k$  is defined as:

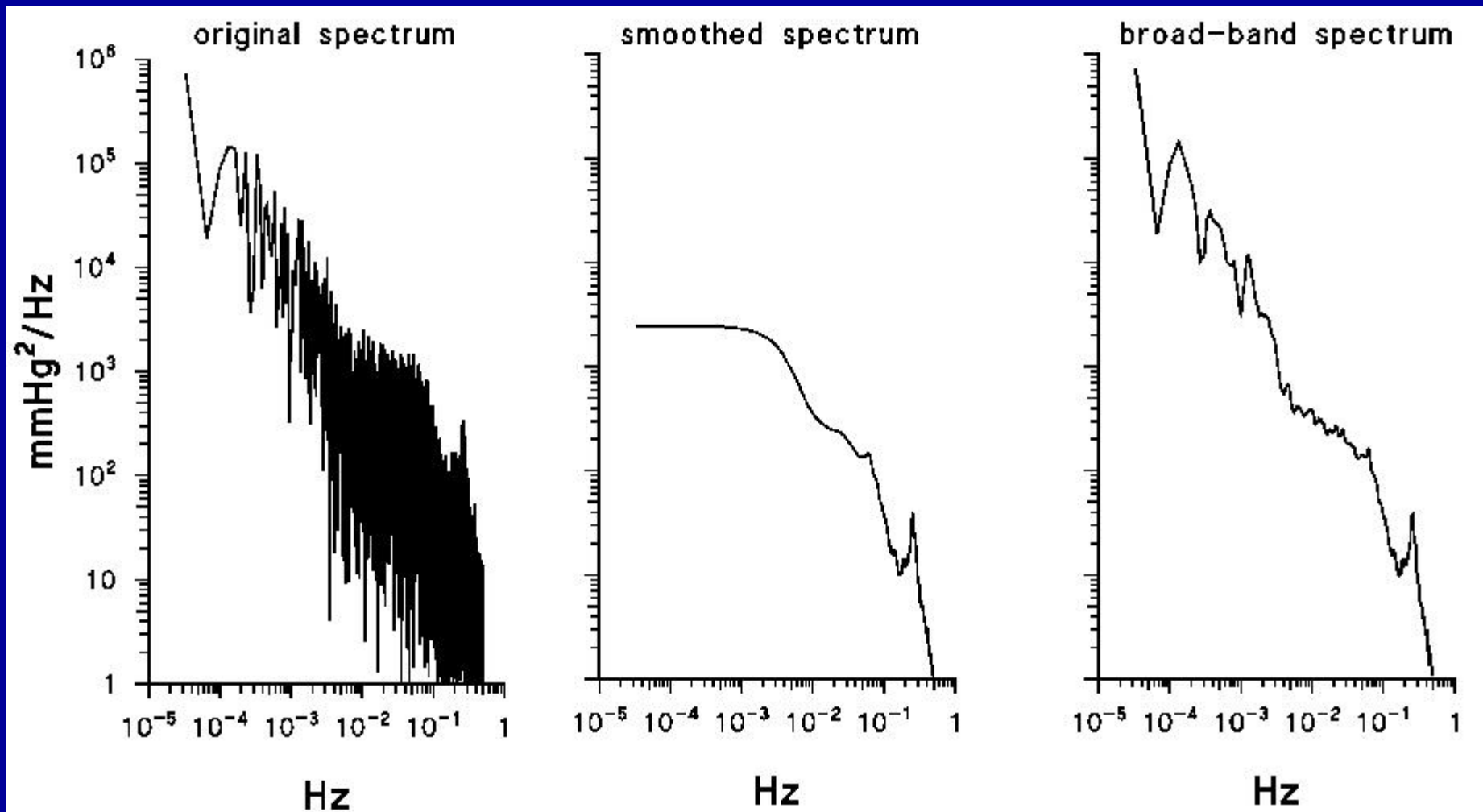
$$R_{xx}(k) = \frac{1}{N-k} \sum_{n=1}^{N-k} x(n)x(n+k)$$

The value of the autocorrelation function at lag 0 is the power of  $x(n)$ , or its variance if the mean value of  $x(n)$  is zero:

$$R_{xx}(0) = \frac{1}{N} \sum_{n=1}^N x(n)^2$$

Moreover,  $\sqrt{R_{xx}(\infty)}$  is the mean value for random processes.

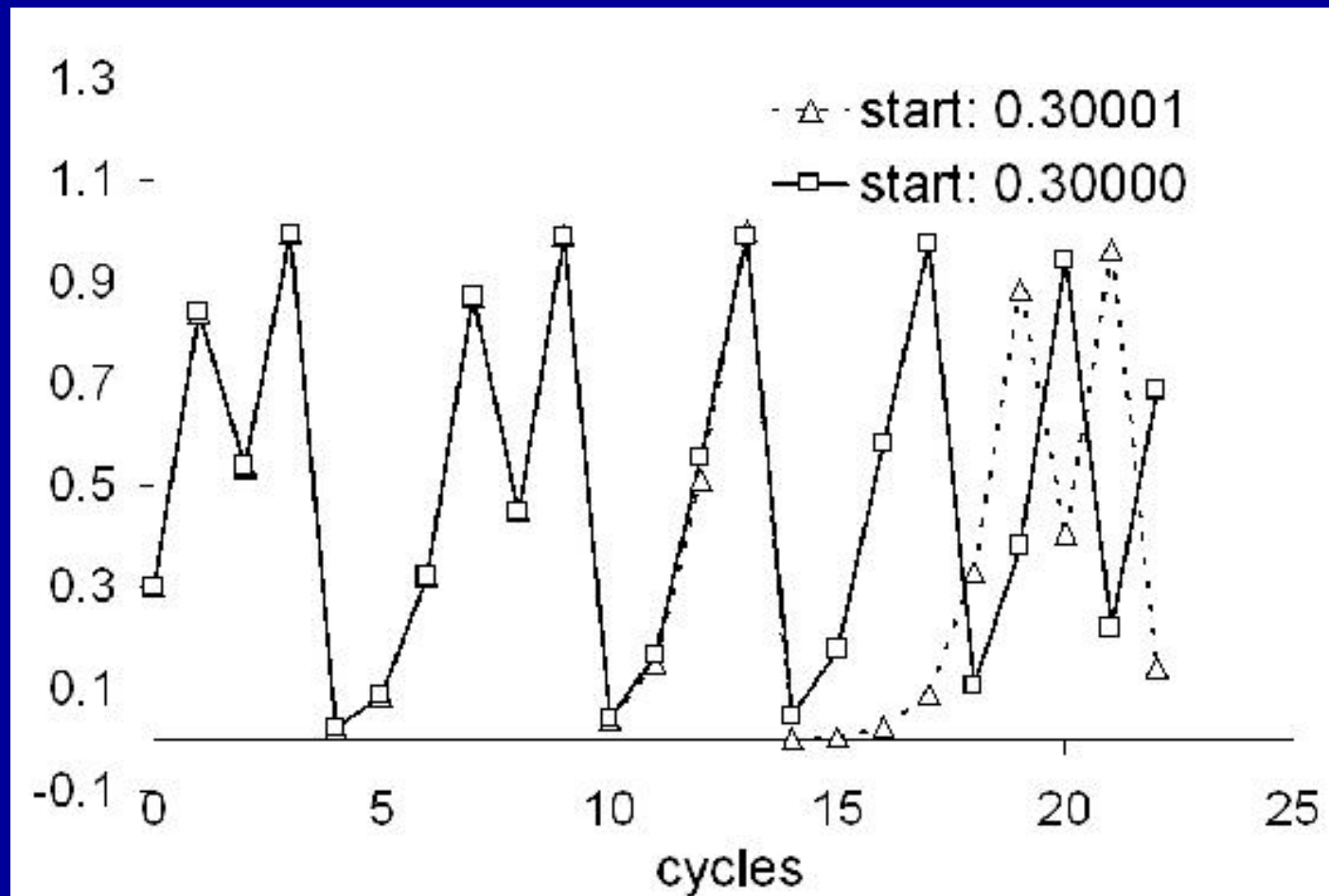
# Broad Band Smoothing



*Left: unsmoothed FFT spectrum of blood pressure from a 8-h recording: this spectrum is characterized by a very high frequency resolution, but also by a very high estimation variance. Centre: the same spectrum smoothed by a moving average filter of order 250 (i.e., average over 250 adjacent spectral lines). Estimation variance is largely reduced, but the frequency resolution dramatically worsens and important spectral details may be lost at the lower frequencies. Right: broad-band spectrum obtained from the raw FFT spectrum by averaging adjacent spectral lines: in this case the number of lines to average increases with the frequency from 1 to 250. The desired reduction of the estimation variance is obtained at the highest frequencies preserving the original frequency resolution at the lowest frequencies.*



# Chaos

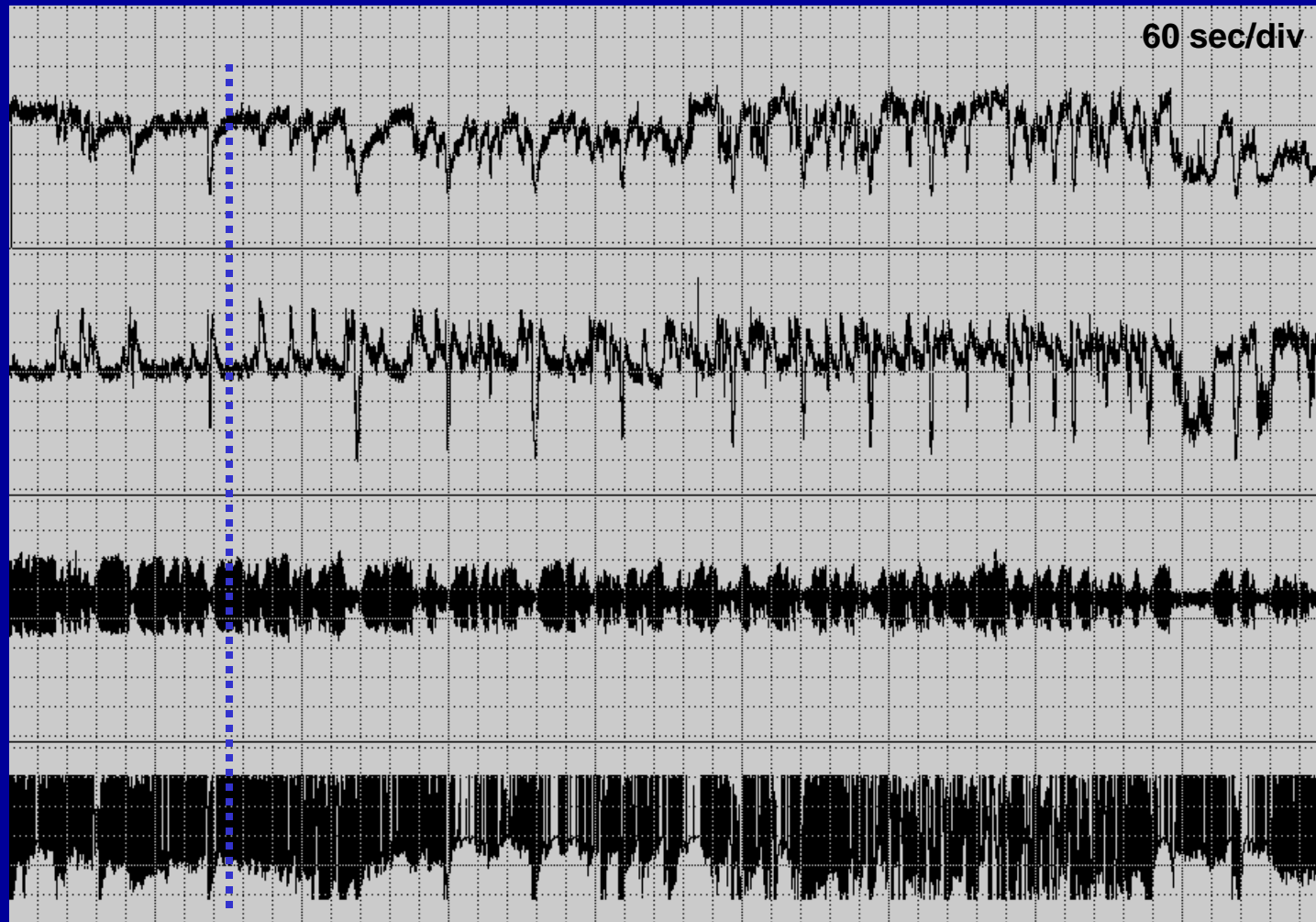


*Sensitivity to initial conditions. Small changes in initial conditions lead to totally different behaviour patterns after a certain time (here 14 cycles). This sensitivity to initial conditions may be quantified by means of the largest Lyapunov exponent.*



# Another Type of Experiment

# Experiment



Blood Flow  
Finger 2

Blood Flow  
Finger 4

PPG

RESP

60 sec/div

← 45 minutes →

Dr. HN Mayrovitz

# Experiment



**Blood Flow  
Finger 2**

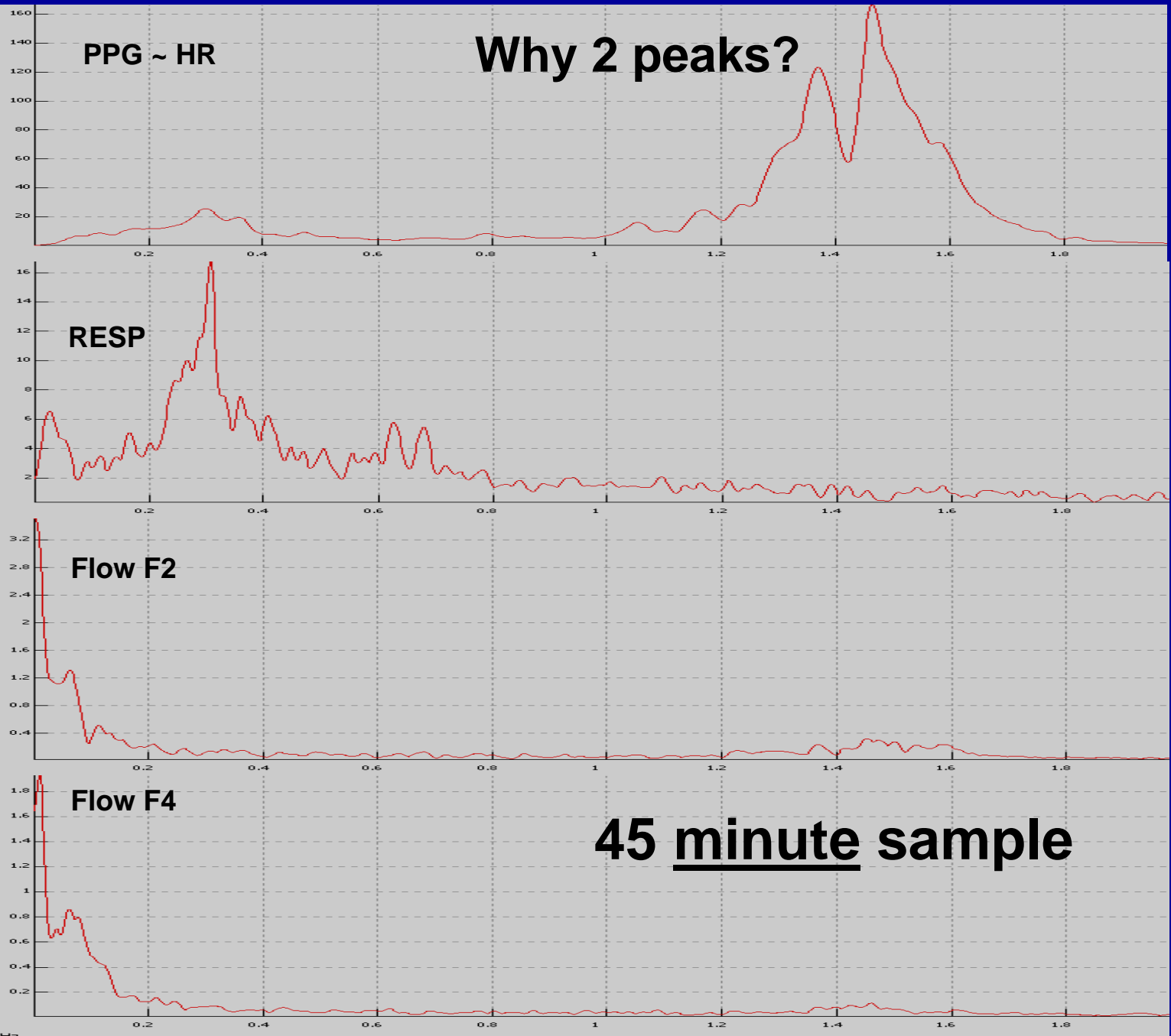
**Blood Flow  
Finger 4**

**PPG**

**RESP**

← 45 seconds →

Dr. HN Mayrovitz

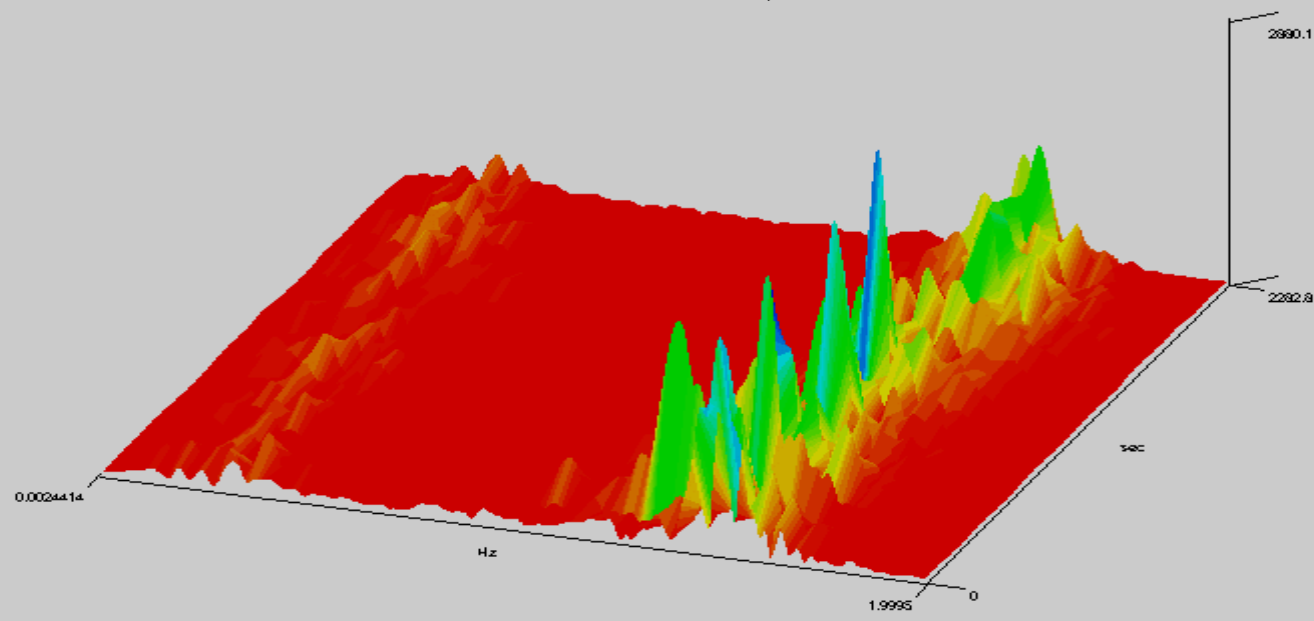
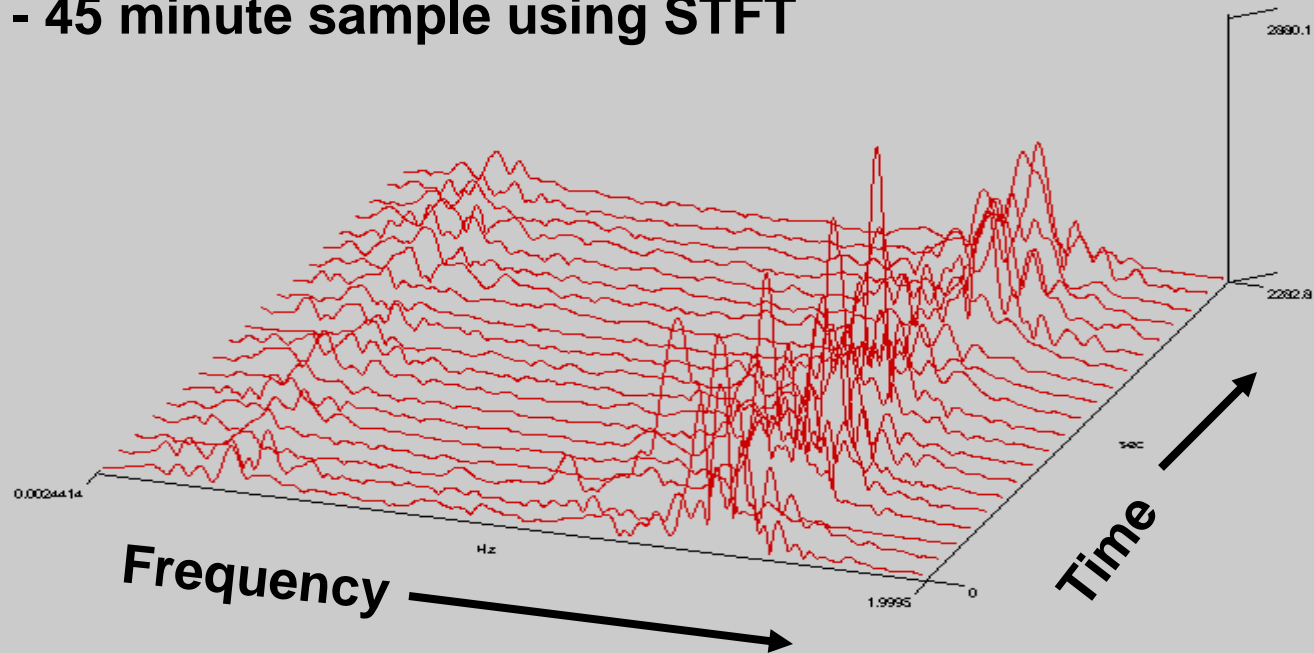


**45 minute sample**

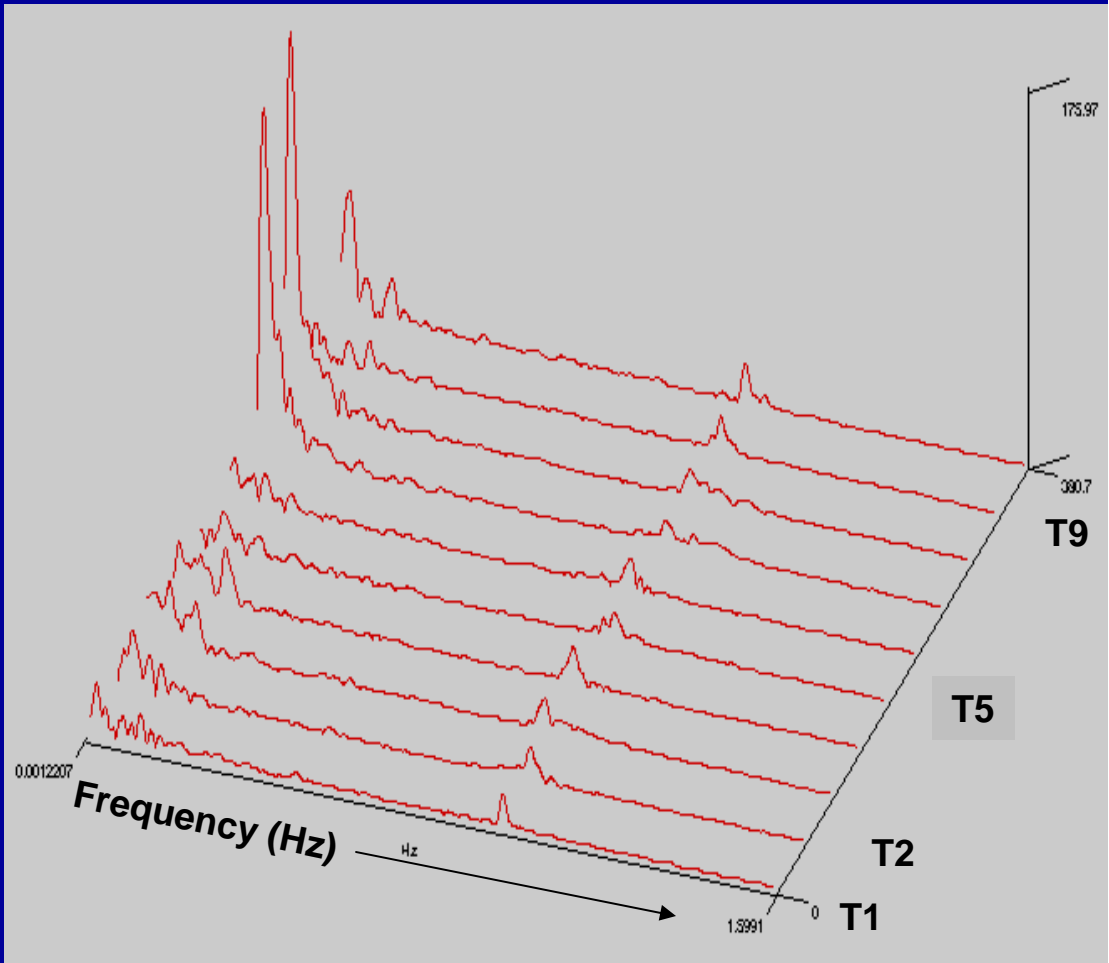
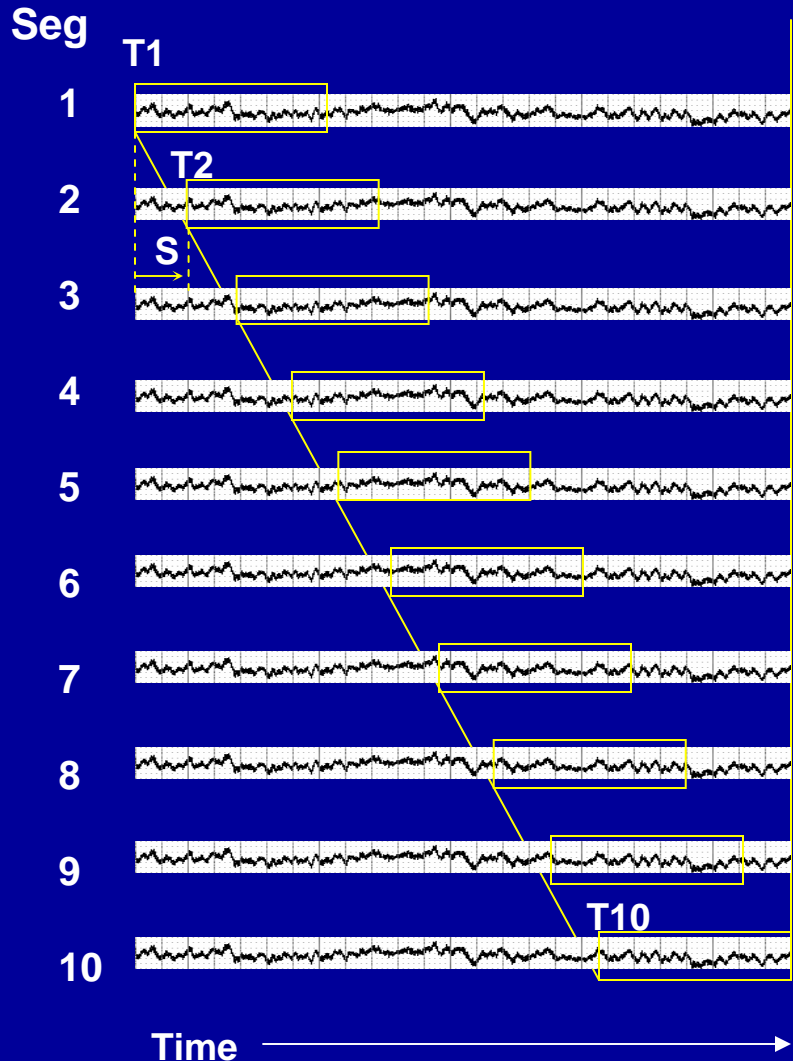
**Physiological signals whose spectral  
content changes with time**

**Principle of STFT  
Short Time Fourier Transform**

# PPG - 45 minute sample using STFT



# Principles of Short Time Fourier Transform Analysis



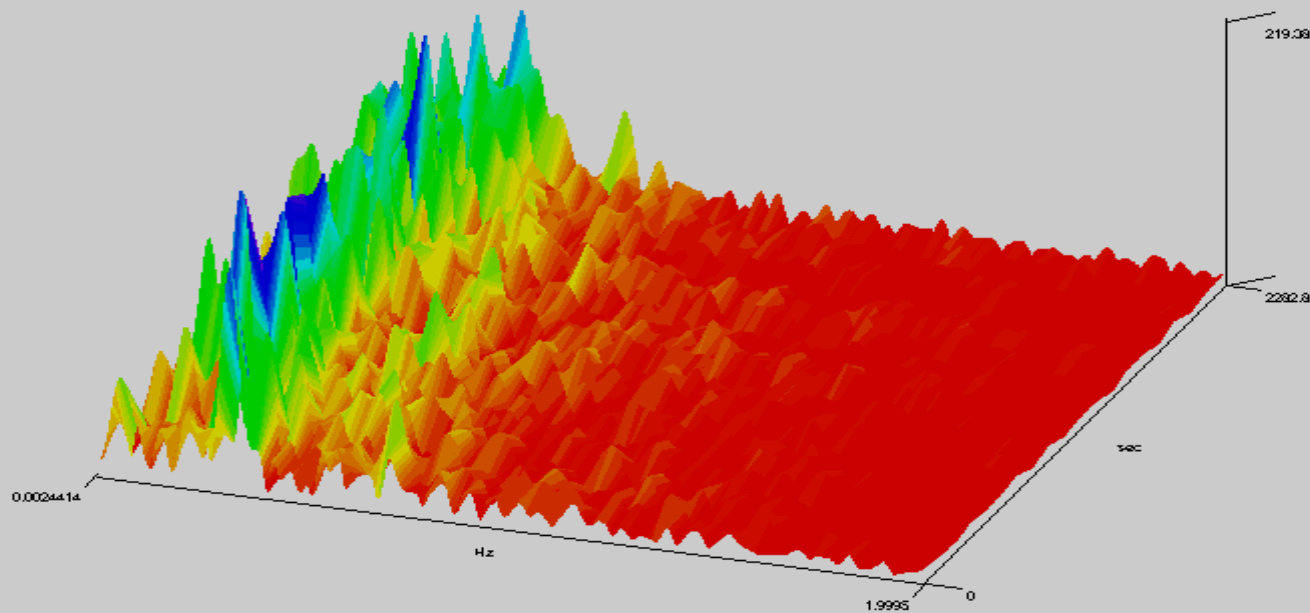
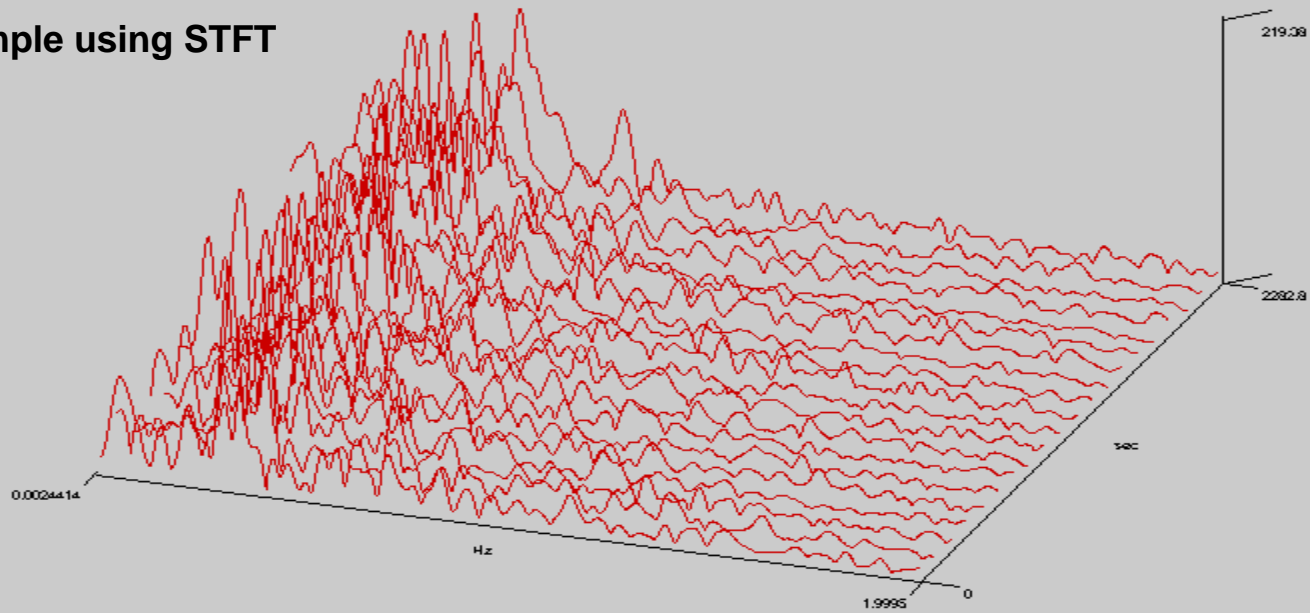
$T_{total} = 20 \text{ minutes} = 1200 \text{ sec}$ ,  $F_s = 20 \text{ s/sec}$   
 $N_{precision} = 16384 = 16384/20 = 819.2 \text{ sec}$   
 $F_{precision} = (1/819.2) = 0.0012 \text{ Hz}$

$$\begin{aligned}
 T_{10} &= T_{total} - N_{precision}/F_s \\
 &= 1200 - 819.2 = 380.7 \text{ sec} \\
 &= (N_{segs}-1) \times S = 9 \times 846/20 = 9 \times 42.3 \text{ sec} = 380.7 \text{ sec}
 \end{aligned}$$

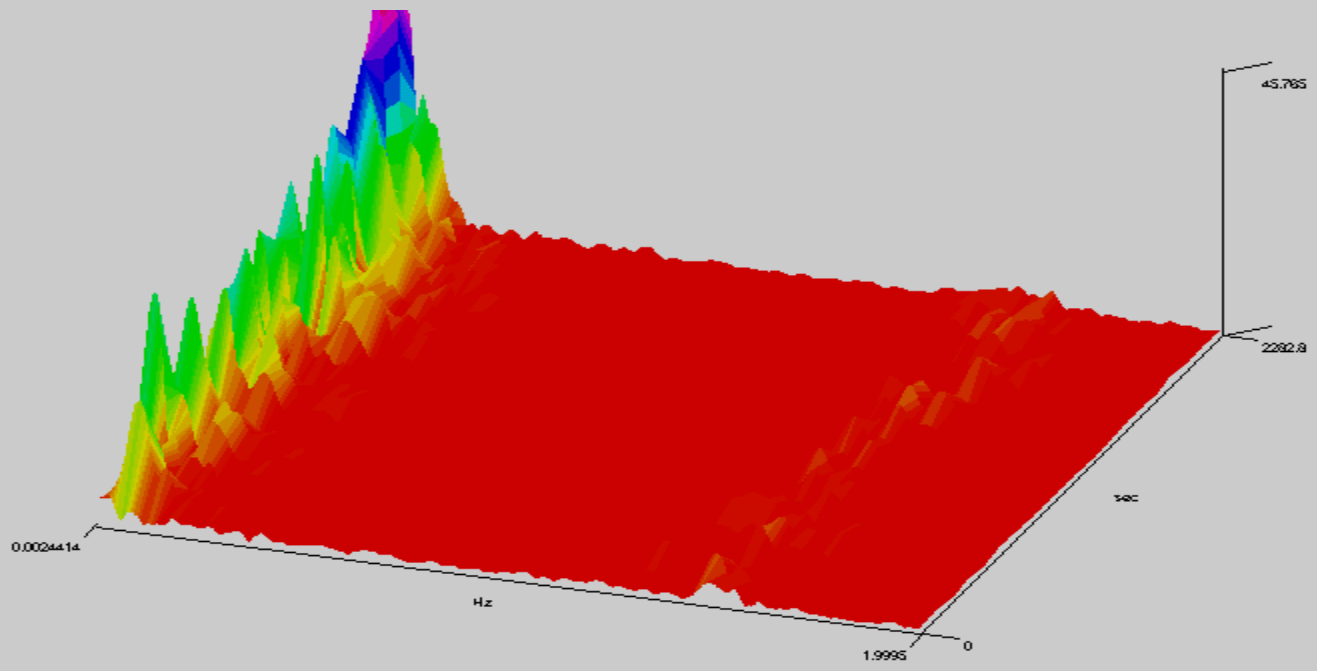
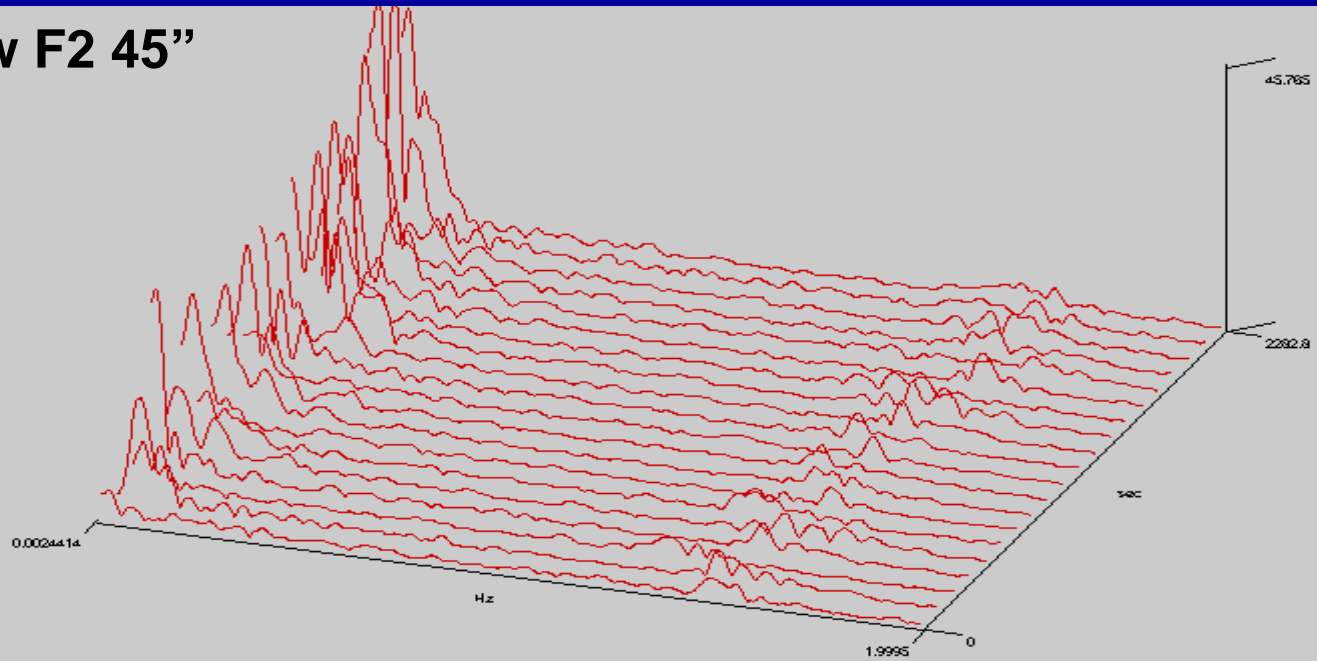


# RESP

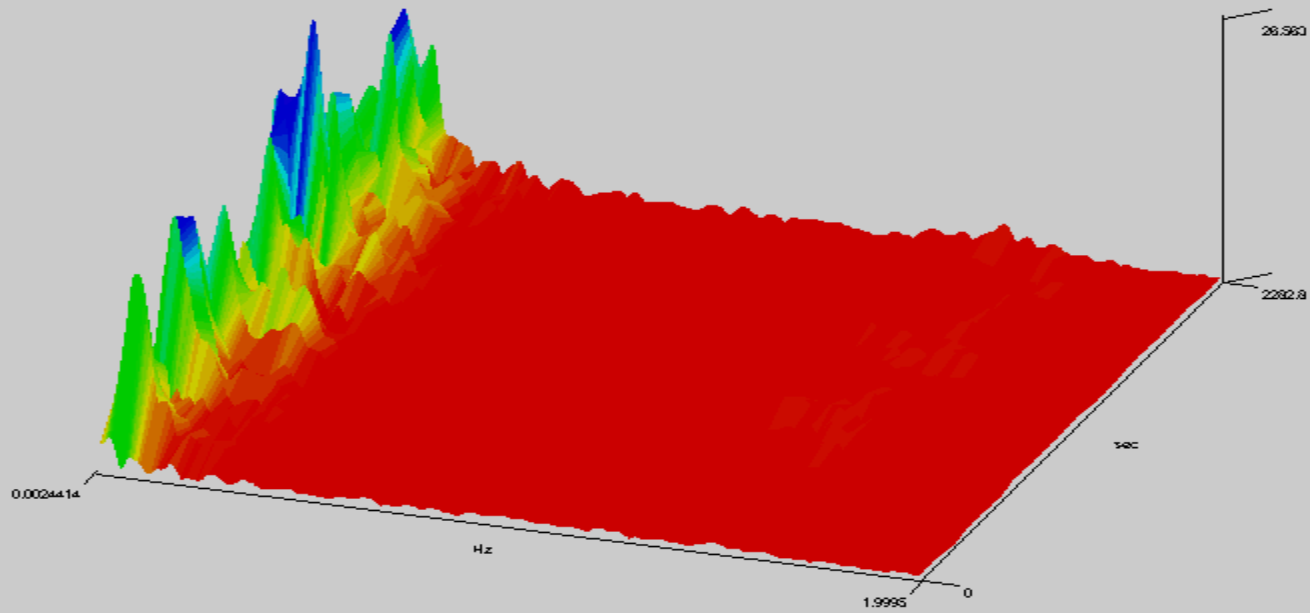
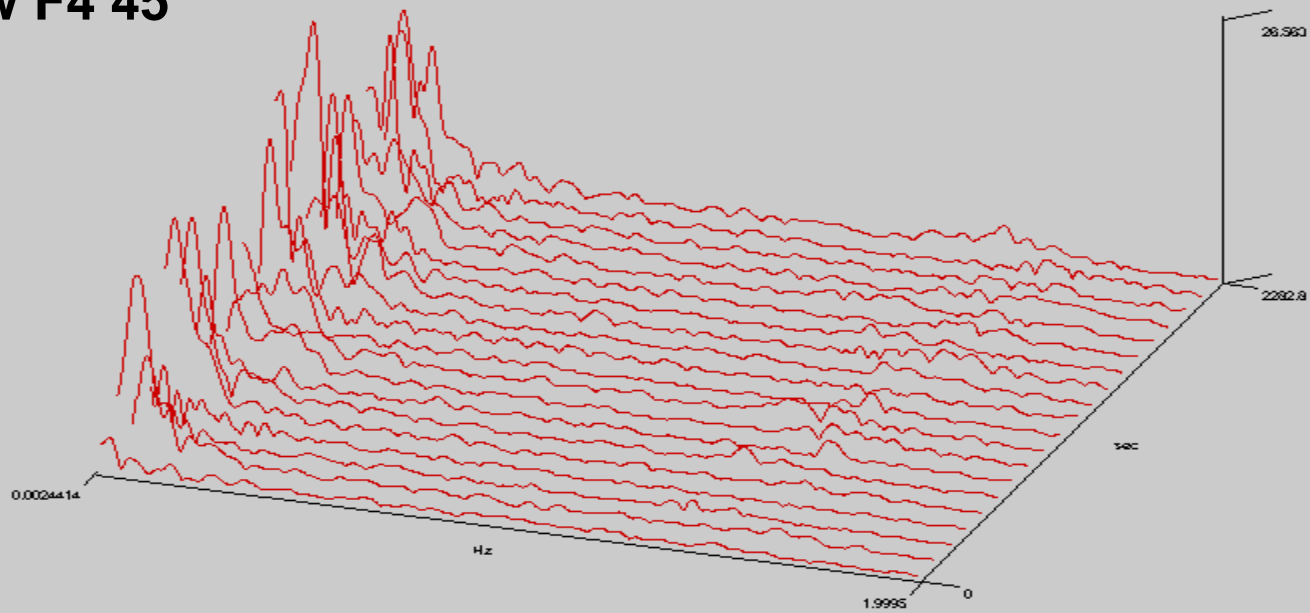
45" sample using STFT



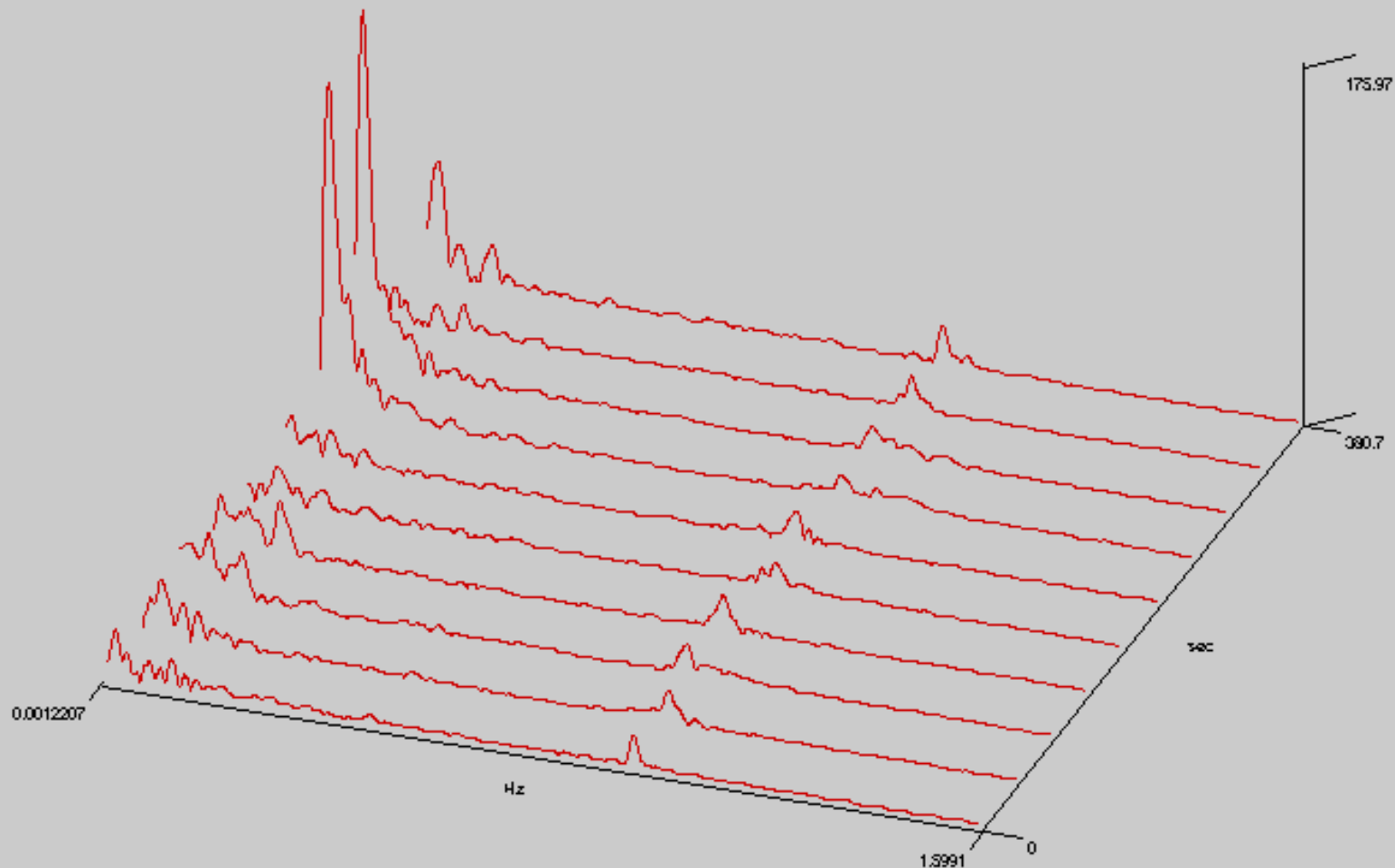
# Flow F2 45"



# Flow F4 45"



**HNM: 20" moor signal at 20 s/sec = 24000 pts on left hand =  $24000/20=1200$  sec  
precision=16384, #seg=10 therefore step =1756  
precision  $\sim 16384/20 = 819.2$  sec; step  $\sim 846/20 = 42.3$  sec**



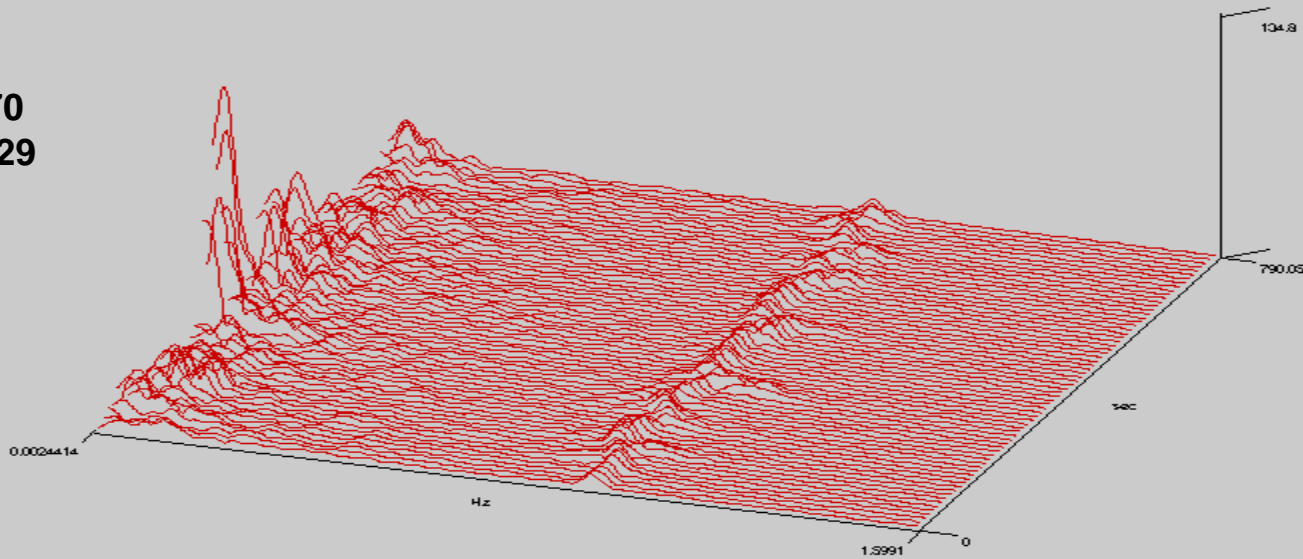
**Precision=number of points per spectrum**

**Step = S = number of points from start of one spectrum to start of the next**

HNM: 20" moor signal at 20 s/s = 24000 pts on LH, both with precision = 8192, Fs = 20

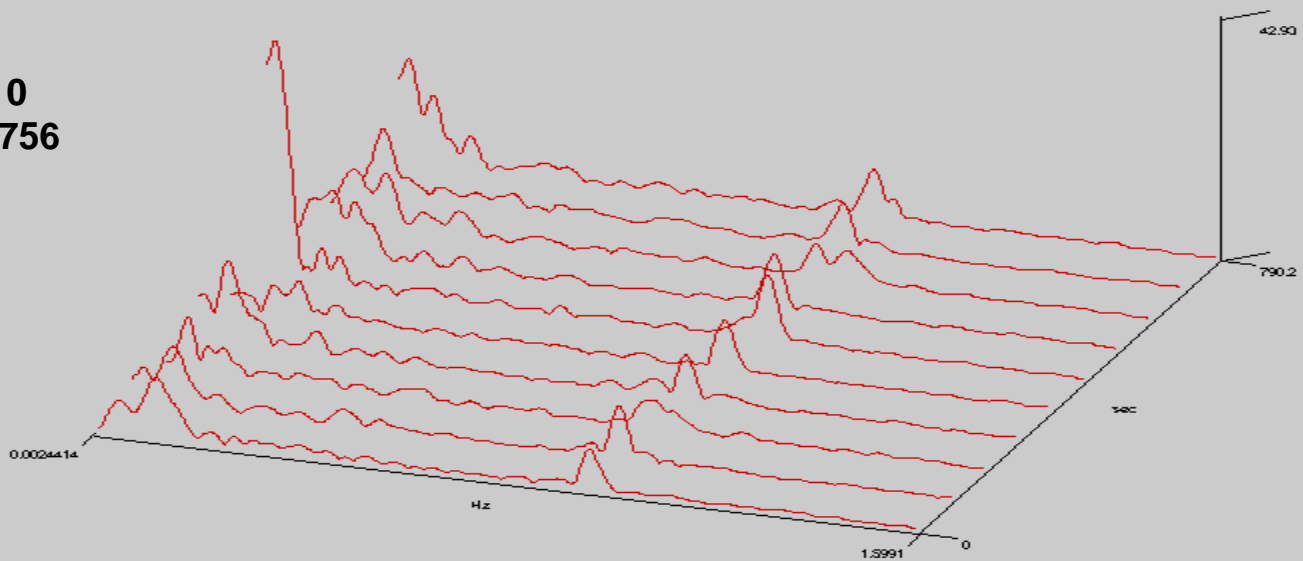
A

#seg=70  
step=229



B

#seg=10  
step=1756



Variable	Units	Description	Frequency Range
<hr/> <b>Analysis of Short-term Recordings (5 min)</b> <hr/>			
5-min total power	ms <sup>2</sup>	The variance of NN intervals over the temporal segment	≈≤0.4 Hz
VLF	ms <sup>2</sup>	Power in VLF range	≤0.04 Hz
LF	ms <sup>2</sup>	Power in LF range	0.04-0.15 Hz
LF norm	nu	LF power in normalized units LF/(total power-VLF)x100	
HF	ms <sup>2</sup>	Power in HF range	0.15-0.4 Hz
HF norm	nu	HF power in normalized units HF/(total power-VLF)x100	
LF/HF		Ratio LF [ms <sup>2</sup> ]/HF[ms <sup>2</sup> ]	
<hr/> <b>Analysis of Entire 24 Hours</b> <hr/>			
Total power	ms <sup>2</sup>	Variance of all NN intervals	≈≤0.4 Hz
ULF	ms <sup>2</sup>	Power in the ULF range	≤0.003 Hz
VLF	ms <sup>2</sup>	Power in the VLF range	0.003-0.04 Hz
LF	ms <sup>2</sup>	Power in the LF range	0.04-0.15 Hz
HF	ms <sup>2</sup>	Power in the HF range	0.15-0.4 Hz
α		Slope of the linear interpolation of the spectrum in a log-log scale	≈≤0.04 Hz

Variable	Units	Normal Values (mean±SD)
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Time Domain Analysis of Nominal 24 hours<sup>181</sup>

SDNN	ms	141±39
SDANN	ms	127±35
RMSSD	ms	27±12
HRV triangular index		37±15

Spectral Analysis of Stationary Supine 5-min Recording

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Total power	ms <sup>2</sup>	3466 ±1018
LF	ms <sup>2</sup>	1170±416
HF	ms <sup>2</sup>	975±203
LF	nu	54±4
HF	nu	29±3
LF/HF ratio		1.5-2.0